



The Sharp EL-531 calculator may be used on this test.  
Show all of your work in the space provided.  
The number of marks for each question is indicated in brackets.

Mark:

25

1. Find the limit and show your work. If the limit does not exist, then answer  $\infty$  or  $-\infty$  if applicable.

(a)  $\lim_{x \rightarrow \infty} \frac{4 + 5x - 2x^2}{3 - 11x + 7x^2}$

[2]

(b)  $\lim_{x \rightarrow -\infty} \frac{5x^{4/3} - 8x^{1/3} + 1}{2x^{1/3} + 5}$

[2]

2. If  $y = \frac{3}{2x+1}$ , then find the differential  $dy$ .

[2]

3. Use implicit differentiation to find  $\frac{dy}{dx}$  for the curve  $y = \sin xy$ .

[3]

4. Suppose  $f$  is a differentiable function satisfying  $f(10) = 4$  and  $f'(10) = -5$ . Use a linear approximation to approximate  $f(10.02)$ .

[2]

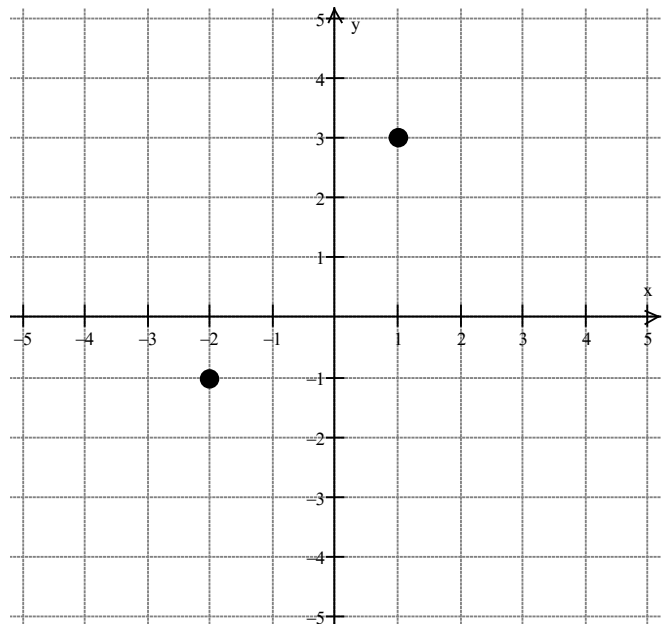
5. Let  $f(x) = \frac{2x^4 + x^2 + 2}{x^4 + 1}$ . Its derivative is  $f'(x) = \frac{-2x^5 + 2x}{(x^4 + 1)^2}$ . Find the open intervals on which  $f$  is increasing or decreasing and find the relative extrema of  $f$ .

[4]

6. Sketch the graph of a **continuous** function  $f$  that passes through the indicated points and that satisfies all of the following properties. Your graph should clearly show the increasing, decreasing and concave structure of  $f$ .

$f'(-2) = 0$
$f'(1)$ is undefined

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
Sign of $f'(x)$	-	+	-
Sign of $f''(x)$	+	+	-

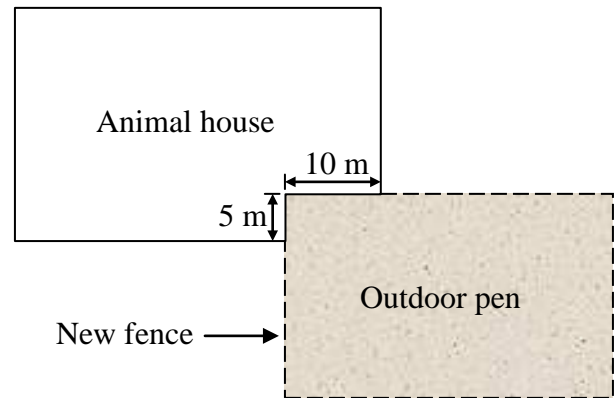


[2]

7. Use Newton's Method with an initial estimate of  $x_1 = 1$  to approximate a solution of the equation  $x - \cos x = 0$ . Round your answer to as many decimal places as your Sharp EL-531 calculator will give you and list the values of all your estimates  $x_1, x_2, x_3$ , etc.

[3]

8. A zookeeper needs to add a rectangular outdoor pen to an animal house with a corner notch as show in the figure. If 85 m of new fence is available, what dimensions of the pen will maximize its area? No fence will be used along the walls of the animal house. Use calculus (derivatives) to find the optimal dimensions and be sure to verify that the dimensions you find do in fact maximize the area of the pen.



[5]