CAMOSUN

Mathematics 100 Test #2A **Name:** SOLUTIONS

Instructor: George Ballinger **Section:** ________
Term: Fall, 2017

The Sharp EL-531 calculator may be used on this test.
You may not use L'Hôpital's Rule when evaluating limits.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.

Mark: $\frac{\quad}{25}$

1. Fill in the blanks with meaningful answers. No justification is required.

   (a) The formula for Newton's Method is $x_{n+1} =$ $x_n - \dfrac{f(x_n)}{f'(x_n)}$.

   (b) The conclusion of Rolle's Theorem states that there is some c in (a,b) such that $f'(c) = 0$.

   (c) If c is a critical number of f then $f'(c)$ is either undefined or zero.

[3]

   (d) If f' is increasing, then the graph of f is said to be Concave upward.

   (e) If f is continuous on $[a,b]$ and $f'(x) < 0$ for all x in (a,b), then f is decreasing on $[a,b]$.

   (f) The differential of $V = \frac{4}{3}\pi r^3$ is $dV =$ $4\pi r^2\,dr$.

2. Find the limit and show your work. If the limit does not exist, then answer ∞ or $-\infty$ if applicable.

   (a) $$\lim_{x\to\infty} \frac{-9x^2+6x}{7x^2+7} = \lim_{x\to\infty} \frac{-9+\frac{6}{x}}{7+\frac{7}{x^2}} = -\frac{9}{7}$$

   ($\frac{6}{x} \to 0$, $\frac{7}{x^2} \to 0$)

[1]

   (b) $$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{-\sqrt{x^2}} = \lim_{x\to-\infty} -\sqrt{\frac{x^2+1}{x^2}} = \lim_{x\to-\infty} -\sqrt{1+\frac{1}{x^2}}$$

   $$= -\sqrt{1+0} = -1$$

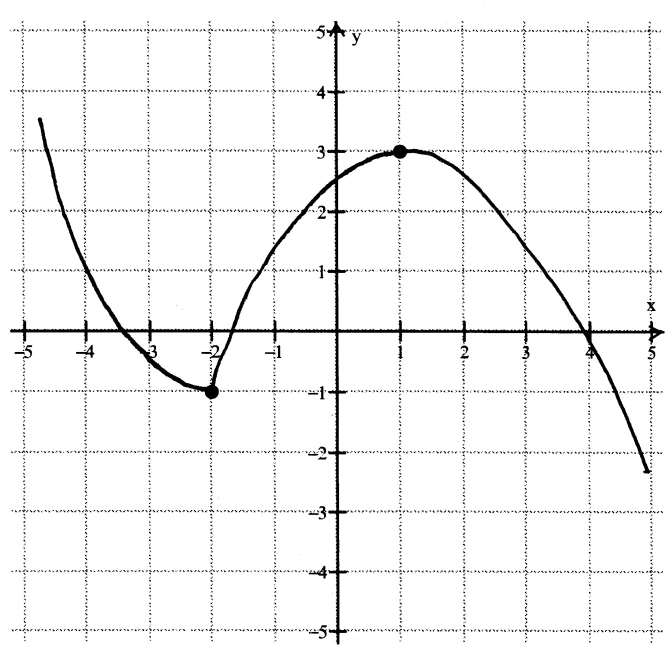
[2]

3. Sketch the graph of a **continuous** function f that passes through the indicated points and that satisfies all of the following properties. Your graph should clearly show the increasing, decreasing and concave structure of f.

$f'(-2)$ is undefined
$f'(1) = 0$

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
Sign of $f'(x)$	− ↘	+ ↗	− ↘
Sign of $f''(x)$	+ ∪	− ∩	− ∩

[2]



4. Find the absolute maximum and minimum values of $f(x)=\frac{1}{2}\sin 2x-\sin x$ on the interval $[0,\pi]$.

$f'(x) = \cos 2x - \cos x = 2\cos^2 x - 1 - \cos x = 2\cos^2 x - \cos x - 1$

$= (2\cos x + 1)(\cos x - 1)$

$f'(x) = 0 \Rightarrow \cos x = -\frac{1}{2}$ or $\cos x = 1$

$\therefore x = \frac{2\pi}{3}$ $\quad x = 0$

[5]

$f(0) = 0$ ← MAX

$f(\frac{2\pi}{3}) = \frac{1}{2}\sin\frac{4\pi}{3} - \sin\frac{2\pi}{3} = \frac{1}{2}(-\frac{\sqrt{3}}{2}) - \frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{4}$ ← MIN

$f(\pi) = 0$ ← MAX

Max value of f is 0

and min value of f is $-\frac{3\sqrt{3}}{4} \approx -1.3$

5. Find the slope of the tangent line to the curve $xy=\sin(x-y)$ at the origin.

$xy' + 1\cdot y = \cos(x-y)(1-y')$

At $(0,0)$, $0\cdot y' + 1\cdot 0 = \cos(0-0)(1-y')$

[3]

$0 = 1\cdot(1-y')$

$0 = 1-y'$

$y' = 1$

Slope is 1.

6. Given the following function and its first two derivatives,

$$f(x)=\frac{x-1}{(x+3)^3}, \qquad f'(x)=\frac{-2(x-3)}{(x+3)^4}, \qquad f''(x)=\frac{6(x-5)}{(x+3)^5}.$$

find the intervals on which the graph of f is concave upward or concave downward and find the coordinates of any inflection points.

$f''(x) = 0$ at $x = 5$ $\qquad f(5) = \frac{4}{512} = \frac{1}{128}$

$f''(x)$ undef. at $x=-3$ (discont. of f)

[3]

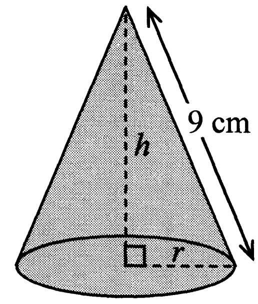
Intervals	$(-\infty,-3)$	$(-3,5)$	$(5,\infty)$
Sign of f''	+	—	+
f	∪	∩	∪

Graph is concave up on $(-\infty,-3)$ and $(5,\infty)$

Graph is concave down on $(-3,5)$

$(5,\frac{1}{128})$ is an inflection point.

7. Find the maximum volume of a right circular cone if its slant height is 9 cm as shown in the figure. Use the Second Derivative Test to verify that your answer is indeed a maximum.



Max $V = \frac{1}{3}\pi r^2 h$

Subject to $r^2 + h^2 = 9^2$

$r^2 = 81 - h^2$

$\therefore V = \frac{1}{3}\pi(81-h^2)h = \frac{1}{3}\pi(81h - h^3)$, $0 \le h \le 9$

$V' = \frac{1}{3}\pi(81 - 3h^2) = \pi(27 - h^2)$

[6]

$V' = 0$ when $h^2 = 27 \Rightarrow h = 3\sqrt{3}$ or $\underbrace{-3\sqrt{3}}_{\text{discard}}$

$V'' = -2\pi h$

at $h = 3\sqrt{3}$, $V'' = -6\sqrt{3}\pi < 0$ $\therefore$ Max

when $h = 3\sqrt{3}$. $V = \frac{1}{3}\pi\left(81 - (3\sqrt{3})^2\right)(3\sqrt{3})$

$= \frac{1}{3}\pi(54)(3\sqrt{3}) = 54\sqrt{3}\pi$

Max Volume is $54\sqrt{3}\pi \text{ cm}^3 \approx 293.8 \text{ cm}^3$