CAMOSUN

Name: SOLUTIONS

Mark: $\overline{\quad}$ 25

MATH 100 (Fall, 2022)

Test 2A

1. Let $f(x) = 2\cos x - x$.

   (a) (3 marks) If Newton's Method were used to approximate a zero of $f(x)$ using an initial approximation of $x_1 = 1$, then compute the next approximation x_2. Round your answer to four decimal places.

   $f'(x) = -2\sin x - 1$

   $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{2\cos 1 - 1}{-2\sin 1 - 1} \approx 1.0300$

   (b) (3 marks) Verify that f satisfies the conditions of the Mean Value Theorem on the closed interval $[0, 2\pi]$, and find all values of c in the open interval $(0, 2\pi)$ that satisfy the Mean Value Theorem formula.

   f is continuous on $[0,2\pi]$ and differentiable on $(0,2\pi)$

   $\therefore$ conditions of MVT are satisfied.

   $\frac{f(2\pi) - f(0)}{2\pi - 0} = \frac{(2-2\pi) - 2}{2\pi} = \frac{-2\pi}{2\pi} = -1$

   $f'(c) = -1 \Rightarrow -2\sin c - 1 = -1 \Rightarrow -2\sin c = 0 \Rightarrow \sin c = 0$

   $\Rightarrow c = \pi$ in $(0, 2\pi)$.

2. (3 marks) Find dy/dx given the equation $x \tan y = y \tan x$.

$$x\sec^2 y \cdot y' + 1\cdot \tan y = y\sec^2 x + y'\tan x$$

$$(x\sec^2 y - \tan x)\, y' = y\sec^2 x - \tan y$$

$$y' = \frac{y\sec^2 x - \tan y}{x\sec^2 y - \tan x}$$

3. (4 marks) Consider the function $f(x) = \dfrac{5x^2}{(x-1)^4}$, whose derivative is $f'(x) = -\dfrac{10x(x+1)}{(x-1)^5}$. Find the intervals on which f is increasing or decreasing and find the coordinates of all critical points of f. Classify each critical point as a relative maximum, relative minimum or neither.

$f'(x) = 0$ at $x = 0$, $x = -1$ ← critical values

$f'(x)$ undefined at $x = 1$ ← discontinuity of f (V.A.)

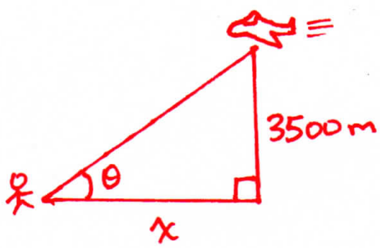
$f(0) = 0$ and $f(-1) = \frac{5}{16}$

Intervals	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
Sign of f'	$+$	$-$	$+$	$-$
f	↗	↘	↗	↘

By FDT $(0,0)$ is a relative min

and $(-1, \frac{5}{16})$ is a relative max

4. (6 marks) An observer on the ground sights an approaching plane flying at an altitude of 3,500 meters and at a constant speed. From his point of view, the plane's angle of elevation is increasing at a rate of 0.01 radians per second when the angle is $\pi/6$. What is the speed of the plane?



Given: $\frac{d\theta}{dt} = 0.01 \frac{rad}{s}$ when $\theta = \frac{\pi}{6}$

Find: $\frac{dx}{dt}$

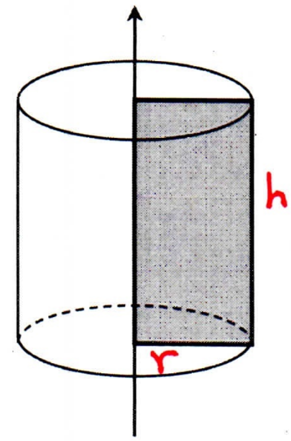
Equation: $\tan\theta = \frac{3500}{x} \Rightarrow x = 3500 \cot\theta$

$$\frac{dx}{dt} = -3500 \csc^2\theta \frac{d\theta}{dt}$$

At $\theta = \pi/6$, $\frac{dx}{dt} = -3500\left(\csc^2 \frac{\pi}{6}\right)(0.01) = (-3500)(4)(0.01) = -140$

$\therefore$ Speed of plane is $140\, m/s$.

5. (6 marks) Consider a rectangle having a perimeter of 42 centimeters. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume? Use the Second Derivative Test to verify that your answer produces a maximum.



Max $V = \pi r^2 h$

Subject to $2h + 2r = 42$

$\Rightarrow h + r = 21$

$\Rightarrow h = 21 - r$

$\therefore V = \pi r^2 (21 - r)$ for $0 \le r \le 21$

$= \pi (21r^2 - r^3)$

$V' = \pi (42r - 3r^2) = 3\pi r (14 - r)$

$V' = 0 \Rightarrow r = 0$ or $\underbrace{r = 14}_{\text{critical value}}$

$V'' = \pi (42 - 6r)$

At $r = 14$, $V'' = \pi(42 - 6(14)) = -42\pi < 0$ $\frown$

$\therefore$ Max by SDT

When $r = 14$, $h = 21 - 14 = 7$

$\therefore$ Dimensions should be 14cm x 7cm with revolution about the shorter edge.