Name: $\qquad$

## Mark:

25

## MATH 100 (Fall, 2022) <br> Test 2A

1. Let $f(x)=2 \cos x-x$.
(a) (3 marks) If Newton's Method were used to approximate a zero of $f(x)$ using an initial approximation of $x_{1}=1$, then compute the next approximation $x_{2}$. Round your answer to four decimal places.
(b) (3 marks) Verify that $f$ satisfies the conditions of the Mean Value Theorem on the closed interval $[0,2 \pi]$, and find all values of $c$ in the open interval $(0,2 \pi)$ that satisfy the Mean Value Theorem formula.
2. (3 marks) Find $d y / d x$ given the equation $x \tan y=y \tan x$.
3. (4 marks) Consider the function $f(x)=\frac{5 x^{2}}{(x-1)^{4}}$, whose derivative is $f^{\prime}(x)=-\frac{10 x(x+1)}{(x-1)^{5}}$. Find the intervals on which $f$ is increasing or decreasing and find the coordinates of all critical points of $f$. Classify each critical point as a relative maximum, relative minimum or neither.
4. (6 marks) An observer on the ground sights an approaching plane flying at an altitude of 3,500 meters and at a constant speed. From his point of view, the plane's angle of elevation is increasing at a rate of 0.01 radians per second when the angle is $\pi / 6$. What is the speed of the plane?
5. (6 marks) Consider a rectangle having a perimeter of 42 centimeters. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume? Use the Second Derivative Test to verify that your answer produces a maximum.

