CAMOSUN COLLEGE

Mathematics 100 Test #1B

Instructor: George Ballinger
Term: Fall, 2013

Name: SOLUTIONS

Section: ______

Mark: $\frac{\quad}{25}$

> The Sharp EL-531 calculator may be used on this test.
> You may not use L'Hôpital's Rule when evaluating limits.
> Show all of your work in the space provided.
> The number of marks for each question is indicated in brackets.

1. Determine whether the function $f(x) = x\sin x$ is even, odd or neither.

[1]

$$f(-x) = -x\sin(-x) = x\sin x = f(x)$$

$\therefore$ f is even

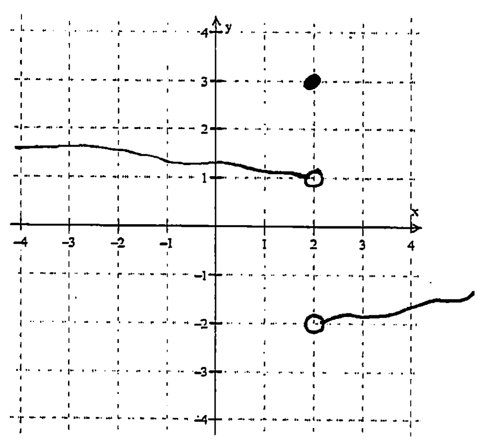
2. Evaluate the one-sided limit $\lim_{x\to 7^+} f(x)$, given $f(x) = \begin{cases} 5-6x, & x \le 7 \\ 2+10x, & x > 7 \end{cases}$.

[1]

$$\lim_{x\to 7^+} (2+10x) = 2+10(7) = 72$$

3. Sketch the graph of a function that is continuous at all $x \ne 2$ and that satisfies the following: $f(2) = 3$, $\lim_{x\to 2^-} f(x) = 1$ and $\lim_{x\to 2^+} f(x) = -2$.

[1]



4. Suppose $1 - x^2 \le f(x) \le \cos x$ for all x. Use the Squeeze Theorem to find $\lim_{x\to 0} f(x)$.

[1]

$\lim_{x\to 0} f(x) = 1$ since $\lim_{x\to 0}(1-x^2) = 1$ and $\lim_{x\to 0} \cos x = 1$

5. What is the value of the following limit? You do not need to show any work.

[1]

$$\underbrace{\lim_{\Delta x \to 0} \frac{\sec(x+\Delta x) - \sec x}{\Delta x}}_{\text{derivative of } \sec x} = \underline{\sec x \tan x}.$$

6. Evaluate the limits and show your work. If they do not exist, then answer ∞ or $-\infty$ if applicable.

(a) $\lim\limits_{x\to\pi}\frac{\sin x}{x}$ $= \frac{0}{\pi} = 0$

[1]

(b) $\lim\limits_{x\to 2}\frac{2x^2-8}{x-2}$ $= \lim\limits_{x\to 2} \frac{2(x^2-4)}{x-2} = \lim\limits_{x\to 2} \frac{2(x+2)\cancel{(x-2)}}{\cancel{x-2}} = \lim\limits_{x\to 2} 2(x+2) = 8$

[1]

(c) $\lim\limits_{x\to 6}\frac{\frac{5}{2x+3}-\frac{1}{3}}{x-6}$ $= \lim\limits_{x\to 6} \frac{15-(2x+3)}{3(2x+3)(x-6)} = \lim\limits_{x\to 6} \frac{-2x+12}{3(2x+3)(x-6)}$

$= \lim\limits_{x\to 6} \frac{-2(x-6)}{3(2x+3)(x-6)} = \lim\limits_{x\to 6} \frac{-2}{3(2x+3)} = \frac{-2}{45}$

[3]

7. Let $f(x)=5-3\sqrt{x+4}$. Use the **limit definition** of the derivative to find $f'(x)$.

$f'(x) = \lim\limits_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim\limits_{\Delta x\to 0} \frac{\left[5-3\sqrt{x+\Delta x+4}\right]-\left[5-3\sqrt{x+4}\right]}{\Delta x}$

$= \lim\limits_{\Delta x\to 0} \frac{-3\left(\sqrt{x+\Delta x+4}-\sqrt{x+4}\right)}{\Delta x} \cdot \frac{\sqrt{x+\Delta x+4}+\sqrt{x+4}}{\sqrt{x+\Delta x+4}+\sqrt{x+4}}$

[3]

$= \lim\limits_{\Delta x\to 0} \frac{-3\left[(x+\Delta x+4)-(x+4)\right]}{\Delta x\left[\sqrt{x+\Delta x+4}+\sqrt{x+4}\right]} = \lim\limits_{\Delta x\to 0} \frac{-3\Delta x}{\Delta x\left[\sqrt{x+\Delta x+4}+\sqrt{x+4}\right]}$

$= \lim\limits_{\Delta x\to 0} \frac{-3}{\sqrt{x+\Delta x+4}+\sqrt{x+4}} = \frac{-3}{\sqrt{x+4}+\sqrt{x+4}} = \frac{-3}{2\sqrt{x+4}}$

8. Use derivative rules to calculate the derivatives of the following functions. Simplify your answers.

(a) $f(x) = \dfrac{x^2 - 2}{11x^2 + 3}$

$$f'(x) = \frac{(11x^2+3)(2x) - (x^2-2)(22x)}{(11x^2+3)^2} = \frac{22x^3 + 6x - 22x^3 + 44x}{(11x^2+3)^2}$$

[2]

$$= \frac{50x}{(11x^2+3)^2}$$

(b) $y = 4\sin^5(8x^3 - 7)$

$$y' = 20\sin^4(8x^3-7) \cdot \cos(8x^3-7) \cdot 24x^2$$

[2]

$$= 480x^2 \sin^4(8x^3-7)\cos(8x^3-7)$$

(c) $h(t) = t \cdot \sqrt[3]{1-t}$ $= t(1-t)^{1/3}$

$$h'(t) = t \cdot \tfrac{1}{3}(1-t)^{-2/3}(-1) + (1-t)^{1/3}(1)$$

$$= -\tfrac{1}{3}t(1-t)^{-2/3} + (1-t)^{1/3}$$

[3]

$$= -\tfrac{1}{3}(1-t)^{-2/3}[t - 3(1-t)]$$

$$= -\tfrac{1}{3}(1-t)^{-2/3}[t - 3 + 3t]$$

$$= \frac{-(4t-3)}{3(1-t)^{2/3}}$$

9. The position of an object in meters after t seconds is given by $s(t) = 8t + \dfrac{9}{t} + 2$, for $t > 1$. Calculate the instantaneous velocity (in m/s) at $t = 3$ seconds.

$$s'(t) = 8 - \frac{9}{t^2}$$

[2]

$$s'(3) = 7$$

7 m/s

10. Prove $\frac{d}{dx}\cos x = -\sin x$.

[3]

$$\frac{d}{dx}\cos x = \lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - \cos x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\cos x (1 - \cos \Delta x) - \sin x \sin \Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left[-\cos x \cdot \overset{0}{\cancel{\frac{1-\cos \Delta x}{\Delta x}}} - \sin x \cdot \overset{1}{\cancel{\frac{\sin \Delta x}{\Delta x}}} \right]$$

$$= (-\cos x)(0) - (\sin x)(1) = -\sin x$$