CAMOSUN

Mathematics 100 Test #1A Name: SOLUTIONS

Instructor: George Ballinger Section: ______

Term: Fall, 2017

The Sharp EL-531 calculator may be used on this test.
You may not use L'Hôpital's Rule when evaluating limits.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.

Mark: $\frac{\quad}{25}$

1. Fill in the blanks with meaningful answers. No justification is required.

[2]

(a) The graph of an odd function is symmetric about the origin.

(b) If a function f is continuous at $x = a$, then $\lim_{x\to a} f(x) =$ $f(a)$.

(c) $f(x) = \dfrac{x}{x^2+7x}$ has a removable discontinuity at $x =$ 0.

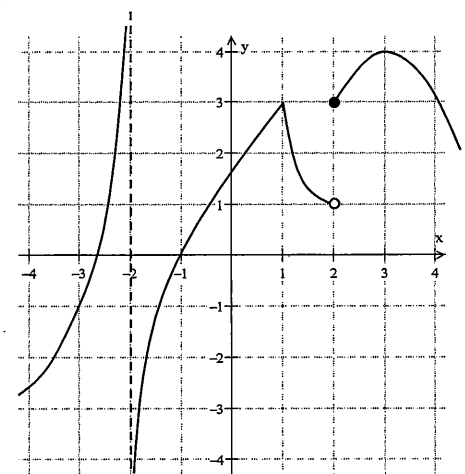
(d) $\displaystyle\lim_{\Delta x\to 0} \frac{\cot(x+\Delta x)-\cot x}{\Delta x} =$ $-\csc^2 x$.

2. For the function $y = f(x)$ whose graph is given, find

[1]

(a) $f(2) =$ 3.

(b) $\lim_{x\to 2^-} f(x) =$ 1.



vertical asymptote

3. Solve $\cos 2\theta = \cos\theta$, where $0 \le \theta \le 2\pi$.

[2]

$2\cos^2\theta - 1 = \cos\theta$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = 1$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\theta = 0, 2\pi$

4. Evaluate the limits and show your work. If they do not exist, then answer ∞ or $-\infty$ if appropriate.

(a) $\lim_{x\to 4}\frac{x^2-16}{2x^2-3x-20}$ $= \lim_{x\to 4} \frac{(x+4)(x-4)}{(2x+5)(x-4)} = \lim_{x\to 4} \frac{x+4}{2x+5} = \frac{8}{13}$

[2]

(b) $\lim_{x\to 0}\frac{1-\cos^2 x}{x\sin x}$ $= \lim_{x\to 0} \frac{\sin^2 x}{x \sin x} = \lim_{x\to 0} \frac{\sin x}{x} = 1$

[2]

(c) $\lim_{x\to 6^+}\frac{2}{6-x}$ $= \frac{2}{0^-} = -\infty$

[1]

5. Prove $\frac{d}{dx}[cf(x)] = cf'(x)$, where c is a constant and $f(x)$ is a differentiable function.

$\frac{d}{dx}[cf(x)] = \lim_{\Delta x\to 0} \frac{cf(x+\Delta x) - cf(x)}{\Delta x} = \lim_{\Delta x\to 0} \frac{c(f(x+\Delta x) - f(x))}{\Delta x}$

$= c \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = c f'(x)$

[2]

6. Use derivative rules to calculate the derivatives of the following functions. Simplify your answers.

(a) $y = \tan(\sec x)$

$$y' = \sec^2(\sec x)\sec x \tan x$$

[2]

(b) $y = \tan x \sec x$

$$y' = \tan x \sec x \tan x + \sec^2 x \sec x$$
$$= \sec x \tan^2 x + \sec^3 x$$

[2]

(c) $g(t) = \dfrac{t}{\sqrt{9-2t^2}}$

$$g'(t) = \frac{\sqrt{9-2t^2}\cdot 1 - t\cdot\frac{-4t}{2\sqrt{9-2t^2}}}{\left(\sqrt{9-2t^2}\right)^2} = \frac{\sqrt{9-2t^2} + \frac{2t^2}{\sqrt{9-2t^2}}}{9-2t^2}$$

[3]

$$= \frac{9-2t^2+2t^2}{(9-2t^2)^{3/2}} = \frac{9}{(9-2t^2)^{3/2}}$$

7. Find the coordinates of the point on the parabola $y = 3x^2 + 9x - 1$ where the slope of the tangent line is parallel to the line $6x + 2y = 3$.

$$2y = -6x + 3$$
$$y = -3x + \frac{3}{2}$$
slope $= -3$

$$y' = 6x + 9$$

Need $y' = -3$

[3]

$$6x + 9 = -3$$
$$6x = -12$$
$$x = -2$$

When $x = -2$, $y = 3(-2)^2 + 9(-2) - 1 = -7$

$\therefore$ Point is $(-2, -7)$

8. Let $f(x) = x^4$. Use the **limit definition** of the derivative to find $f'(x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^4 - x^4}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{x^4} + 4x^3\Delta x + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4 - \cancel{x^4}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\left(4x^3 + 6x^2\Delta x + 4x(\Delta x)^2 + (\Delta x)^3\right)}{\cancel{\Delta x}}$$

[3]

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$= \lim_{\Delta x \to 0} \left(4x^3 + 6x^2\Delta x + 4x(\Delta x)^2 + (\Delta x)^3\right)$$

$$= 4x^3.$$