CAMOSUN

Name: SOLUTIONS

Mark: $\overline{}$ **25**

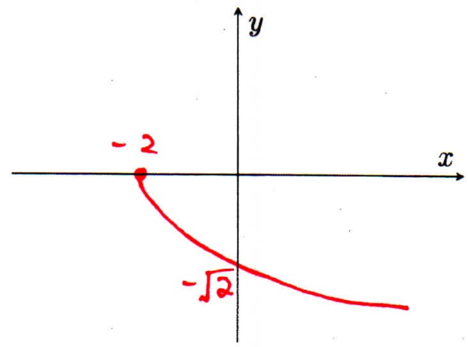
MATH 100 (Fall, 2022)

Test 1B

1. (2 marks) Sketch the graph of $f(x) = -\sqrt{x+2}$ and state its domain and range using interval notation.

domain $[-2, \infty)$

range $(-\infty, 0]$



2. (2 marks) Solve the equation $\sin 2\theta = \cos\theta$ for $0 \le \theta \le 2\pi$.

$2\sin\theta\cos\theta = \cos\theta$

$2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

3. (2 marks) By what theorem can you conclude that the function $f(x) = x^3 - 3x + 1$ has at least one zero in the interval $[-1, 1]$? Verify that all of the conditions of the theorem are satisfied. You do not need to find the zero(s).

Intermediate Value Theorem (IVT)

f is continuous on $[-1,1]$

$f(-1) = 3 > 0$ and $f(1) = -1 < 0$

(i.e. 0 is between $f(-1)$ and $f(1)$.)

4. Evaluate the limits. If they do not exist, then determine whether they are ∞, $-\infty$ or neither.

(a) (1 mark) $\lim_{x\to -5} \frac{4x}{5-x}$ $= \frac{-20}{10} = -2$

(b) (1 mark) $\lim_{x\to 5^-} \frac{4x}{5-x}$ $= \frac{20}{0^+} = \infty$

(c) (1 mark) $\lim_{\theta\to 0} \frac{\sin\theta + 3\theta}{\theta}$ $= \lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta} + 3\right) = 1 + 3 = 4$

(d) (3 marks) $\lim_{x\to 1} \frac{\sqrt{2x-1} - \sqrt{2-x}}{x-1}$

$$= \lim_{x\to 1} \frac{\sqrt{2x-1} - \sqrt{2-x}}{x-1} \cdot \frac{\sqrt{2x-1} + \sqrt{2-x}}{\sqrt{2x-1} + \sqrt{2-x}} = \lim_{x\to 1} \frac{(2x-1) - (2-x)}{(x-1)\left(\sqrt{2x-1} + \sqrt{2-x}\right)}$$

$$= \lim_{x\to 1} \frac{3x-3}{(x-1)\left(\sqrt{2x-1} + \sqrt{2-x}\right)} = \lim_{x\to 1} \frac{3}{\sqrt{2x-1} + \sqrt{2-x}} = \frac{3}{2}$$

5. (3 marks) Differentiate $y = \dfrac{3x^2}{(2x+5)^{3/2}}$ and simplify your answer.

$$y' = \frac{(2x+5)^{3/2}(6x) - 3x^2\left(\frac{3}{2}\right)(2x+5)^{1/2}(2)}{(2x+5)^3}$$

$$= \frac{6x(2x+5)^{3/2} - 9x^2(2x+5)^{1/2}}{(2x+5)^3}$$

$$= \frac{3x(2x+5)^{1/2}\left[2(2x+5) - 3x\right]}{(2x+5)^3}$$

$$= \frac{3x(x+10)}{(2x+5)^{5/2}}$$

6. Let $f(x) = \sec x - \tan x$.

(a) (2 marks) Find the equation of the tangent line to the curve $y = f(x)$ at $x = \pi$. Express your answer in slope-intercept form.

$f(\pi) = -1$ point $(\pi, -1)$

$f'(x) = \sec x \tan x - \sec^2 x$

$f'(\pi) = -1$ (slope).

$y - (-1) = -1(x - \pi)$

$y = -x + \pi - 1$

(b) (2 marks) Find the second derivative of $f(x)$.

$f''(x) = \sec x \cdot \sec^2 x + \sec x \tan x \cdot \tan x - 2\sec x \cdot \sec x \tan x$

$= \sec^3 x + \sec x \tan^2 x - 2\sec^2 x \tan x$

7. The height above the ground, in feet, of a projectile t seconds after being launched into the air is given by

$$s(t) = -16t^2 + 64t + 960.$$

(a) (2 marks) What is the velocity of the projectile 7 seconds after it is launched?

$v(t) = s'(t) = -32t + 64$

$V(7) = -32(7) + 64 = -160$

$\therefore -160 \text{ ft/s}$

(b) (1 mark) How long does it take for the projectile to reach its maximum height?

$V(t) = 0$ at max height

$-32t + 64 = 0 \Rightarrow t = 2$

$\therefore 2 \text{ s}$

8. (3 marks) Use the **limit definition** of a derivative to find $f'(x)$, where $f(x) = 1 + \dfrac{2}{x}$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left(1 + \frac{2}{x+\Delta x}\right) - \left(1 + \frac{2}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{2}{x+\Delta x} - \frac{2}{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x - 2(x+\Delta x)}{x(x+\Delta x)\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2\Delta x}{x(x+\Delta x)\Delta x} = \lim_{\Delta x \to 0} \frac{-2}{x(x+\Delta x)} = \frac{-2}{x^2}$$