

Name: _

 $^{\rm Mark:} \ \overline{25}$

MATH 100 (Fall, 2022) Test 1B

1. (2 marks) Sketch the graph of $f(x) = -\sqrt{x+2}$ and state its domain and range using interval notation.



2. (2 marks) Solve the equation $\sin 2\theta = \cos \theta$ for $0 \le \theta \le 2\pi$.

3. (2 marks) By what theorem can you conclude that the function $f(x) = x^3 - 3x + 1$ has at least one zero in the interval [-1, 1]? Verify that all of the conditions of the theorem are satisfied. You do not need to find the zero(s).

4. Evaluate the limits. If they do not exist, then determine whether they are ∞ , $-\infty$ or neither. (a) (1 mark) $\lim_{x \to -5} \frac{4x}{5-x}$

(b) (1 mark)
$$\lim_{x \to 5^-} \frac{4x}{5-x}$$

(c) (1 mark)
$$\lim_{\theta \to 0} \frac{\sin \theta + 3\theta}{\theta}$$

(d) (3 marks)
$$\lim_{x \to 1} \frac{\sqrt{2x-1} - \sqrt{2-x}}{x-1}$$

5. (3 marks) Differentiate $y = \frac{3x^2}{(2x+5)^{3/2}}$ and simplify your answer.

- 6. Let $f(x) = \sec x \tan x$.
 - (a) (2 marks) Find the equation of the tangent line to the curve y = f(x) at $x = \pi$. Express your answer in slope-intercept form.

(b) (2 marks) Find the second derivative of f(x).

7. The height above the ground, in feet, of a projectile t seconds after being launched into the air is given by

$$s(t) = -16t^2 + 64t + 960.$$

(a) (2 marks) What is the velocity of the projectile 7 seconds after it is launched?

(b) (1 mark) How long does it take for the projectile to reach its maximum height?

8. (3 marks) Use the **limit definition** of a derivative to find f'(x), where $f(x) = 1 + \frac{2}{x}$.