

MATH 100 (Winter, 2026)
Test 2

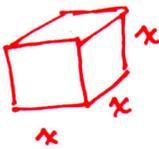
1. (3 marks) Find $\frac{dy}{dx}$ by implicit differentiation.

$$y \cos y = 3x^2 + 5$$

$$(-y \sin y + \cos y) y' = 6x$$

$$y' = \frac{6x}{-y \sin y + \cos y}$$

2. (3 marks) A large block of ice is in the shape of a cube. As it melts, the length of each edge of the cube decreases at a rate of 3 cm/min. At what rate is the ice cube's surface area changing when the length of the edges of the cube is 75 cm?



Given: $\frac{dx}{dt} = -3 \text{ cm/min}$

Find: $\frac{dS}{dt}$ when $x = 75 \text{ cm}$

Equation: $S = 6x^2$ (surface area)

$$\frac{dS}{dt} = 12x \frac{dx}{dt} \quad \text{when } x = 75 \text{ cm, } \frac{dS}{dt} = 12(75 \text{ cm})(-3 \text{ cm/min}) = -2700 \frac{\text{cm}^2}{\text{min}}$$

\therefore Surface area is decreasing at a rate of $2700 \text{ cm}^2/\text{min}$.

3. (3 marks) Find the absolute maximum and minimum values of $f(x) = x - 2\sqrt{x}$ on the interval $[0, 4]$.

$$f'(x) = 1 - \frac{1}{\sqrt{x}} = 0 \Rightarrow \frac{1}{\sqrt{x}} = 1 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1 \text{ critical \#}$$

$$f(0) = 0 \quad (f'(x) \text{ is also undefined at endpoint } x=0)$$

$$f(4) = 0 \quad \leftarrow \text{max}$$

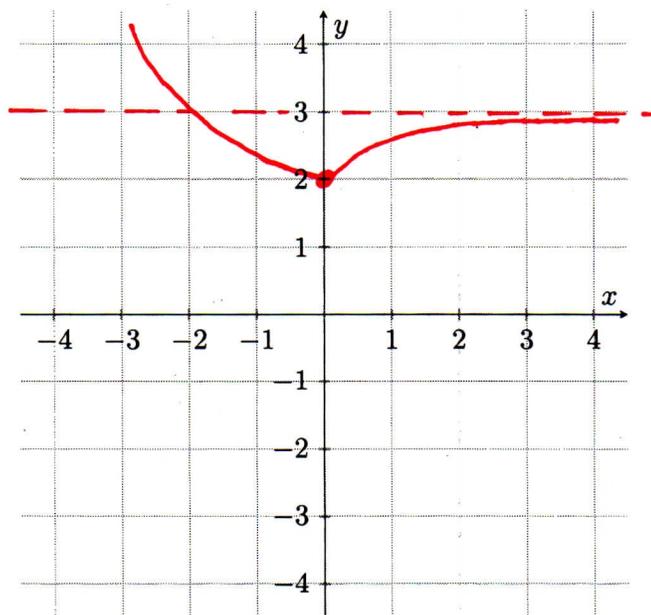
Max is 0 and min is -1

$$f(1) = -1 \quad \leftarrow \text{min}$$

4. (3 marks) Sketch the graph of a **continuous** function that satisfies $f(0) = 2$, $\lim_{x \rightarrow \infty} f(x) = 3$ and whose first and second derivatives have the following signs in the indicated intervals.

	$(-\infty, 0)$	$(0, \infty)$
$f'(x)$	- \rightarrow	+ \nearrow
$f''(x)$	+ \cup	- \cap

\nearrow
H.A.
 $y = 3$



5. Let $f(x) = \sin x + x + 1$.

- (a) (3 marks) If Newton's Method were used to approximate a zero of f using an initial approximation of $x_1 = -0.5$, then compute the next approximation x_2 . Round your answer to four decimal places.

$$f'(x) = \cos x + 1$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\sin x_n + x_n + 1}{\cos x_n + 1}$$

$$x_2 = -0.5 - \frac{\sin(-0.5) + (-0.5) + 1}{\cos(-0.5) + 1} \approx -0.5110$$

- (b) (4 marks) Verify that f satisfies the conditions of the Mean Value Theorem on the interval $[-\pi, \pi]$ and then find the value(s) of c guaranteed to exist by the theorem.

f is continuous on $[-\pi, \pi]$ and differentiable on $(-\pi, \pi)$.

\therefore Conditions of MVT are satisfied

$$\frac{f(\pi) - f(-\pi)}{\pi - (-\pi)} = \frac{(\pi + 1) - (-\pi + 1)}{2\pi} = \frac{2\pi}{2\pi} = 1$$

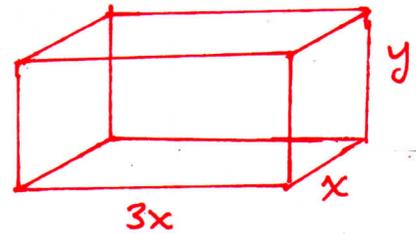
$$f'(x) = 1 \Rightarrow \cos x + 1 = 1 \Rightarrow \cos x = 0 \Rightarrow x = \pm \frac{\pi}{2} \quad (\text{both in } (-\pi, \pi))$$

$$\therefore c = \pm \frac{\pi}{2}$$

6. (6 marks) An open-top box has a rectangular base that is three times as long as it is wide. Find the dimensions of such a box having the smallest surface area if its volume is 144 cm^3 . Use the First Derivative Test to verify that your answer minimizes surface area.

$$\begin{aligned} \text{Minimize } S &= 3x^2 + 2xy + 2(3xy) \\ &= 3x^2 + 8xy \end{aligned}$$

$$\begin{aligned} \text{Subject to } V &= 3x^2y = 144 \\ \Rightarrow y &= \frac{48}{x^2} \end{aligned}$$



$$\therefore S = 3x^2 + 8x \left(\frac{48}{x^2} \right) = 3x^2 + \frac{384}{x} \quad \text{for } x > 0$$

$$S' = 6x - \frac{384}{x^2} = \frac{6x^3 - 384}{x^2} = \frac{6(x^3 - 64)}{x^2} = 0 \Rightarrow x^3 = 64 \Rightarrow x = 4$$

Intervals	$(0, 4)$	$(4, \infty)$
S'	-	+
S	↓	↑

\therefore MIN by FDT

$$\text{When } x=4, \quad 3x=12 \quad \text{and} \quad y = \frac{48}{4^2} = 3$$

\therefore Dimensions are $12 \text{ cm} \times 4 \text{ cm} \times 3 \text{ cm}$.