Mathematics 100 Test #2B Name: SOLUTIONS

Instructor: George Ballinger **Section:** ________
Term: Fall, 2018

The Sharp EL-531 calculator may be used on this test.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.

Mark: ___ / 25

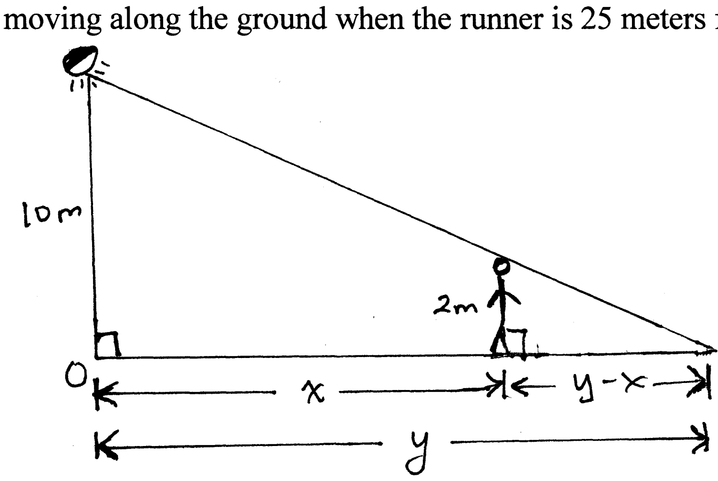
1. If Newton's Method were used to approximate a zero of $f(x) = -x + 4\sqrt{x+1}$ using an initial approximation of $x_1 = 15$, then compute the next approximation x_2.

$$f'(x) = -1 + \frac{2}{\sqrt{x+1}}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 15 - \frac{f(15)}{f'(15)} = 15 - \frac{1}{\left(-\frac{1}{2}\right)} = 17$$

[3]

2. A fugitive who is 2 meters tall runs straight away from a searchlight mounted 10 meters above a point O on the ground. The ground is horizontal and the runner's speed is 8 meters per second. How fast is the shadow of the runner's head moving along the ground when the runner is 25 meters from the point O?



[5]

Given: $\frac{dx}{dt} = 8 \text{ m/s}$

Find: $\frac{dy}{dt}$ when $x = 25\text{m}$

Equation: $\frac{y-x}{2} = \frac{y}{10}$ (similar Δ's)

$$5(y-x) = y$$
$$5y - 5x = y$$
$$4y = 5x \Rightarrow y = \frac{5}{4}x$$

$$\therefore \frac{dy}{dt} = \frac{5}{4}\frac{dx}{dt} = \frac{5}{4}(8) = 10 \text{ m/s}.$$ (independent of x).

$\therefore$ Shadow of head is moving 10 m/s away from point O.

3. Let $f(x) = \frac{1}{2}x + \sin x$ for $0 \le x \le 2\pi$.

(a) Find the open intervals on which f is increasing or decreasing and find the coordinates of all critical points. Classify each critical point as a relative maximum, minimum or neither.

$f'(x) = \frac{1}{2} + \cos x = 0 \Rightarrow \cos x = -\frac{1}{2} \rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$f(\frac{2\pi}{3}) = \frac{\pi}{3} + \frac{\sqrt{3}}{2} \approx 1.9$; $f(\frac{4\pi}{3}) = \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \approx 1.2$

[3]

Intervals	$(0, \frac{2\pi}{3})$	$(\frac{2\pi}{3}, \frac{4\pi}{3})$	$(\frac{4\pi}{3}, 2\pi)$
f'	+	−	+
f	↗	↘	↗

By FDT, local max at $(\frac{2\pi}{3}, \frac{\pi}{3} + \frac{\sqrt{3}}{2})$ and local min at $(\frac{4\pi}{3}, \frac{2\pi}{3} - \frac{\sqrt{3}}{2})$

(b) Find the open intervals on which the graph of f is concave upward or concave downward and find the coordinates of all inflection points.

$f''(x) = -\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$

$f(0) = 0$, $f(\pi) = \frac{\pi}{2} \approx 1.6$, $f(2\pi) = \pi \approx 3.1$

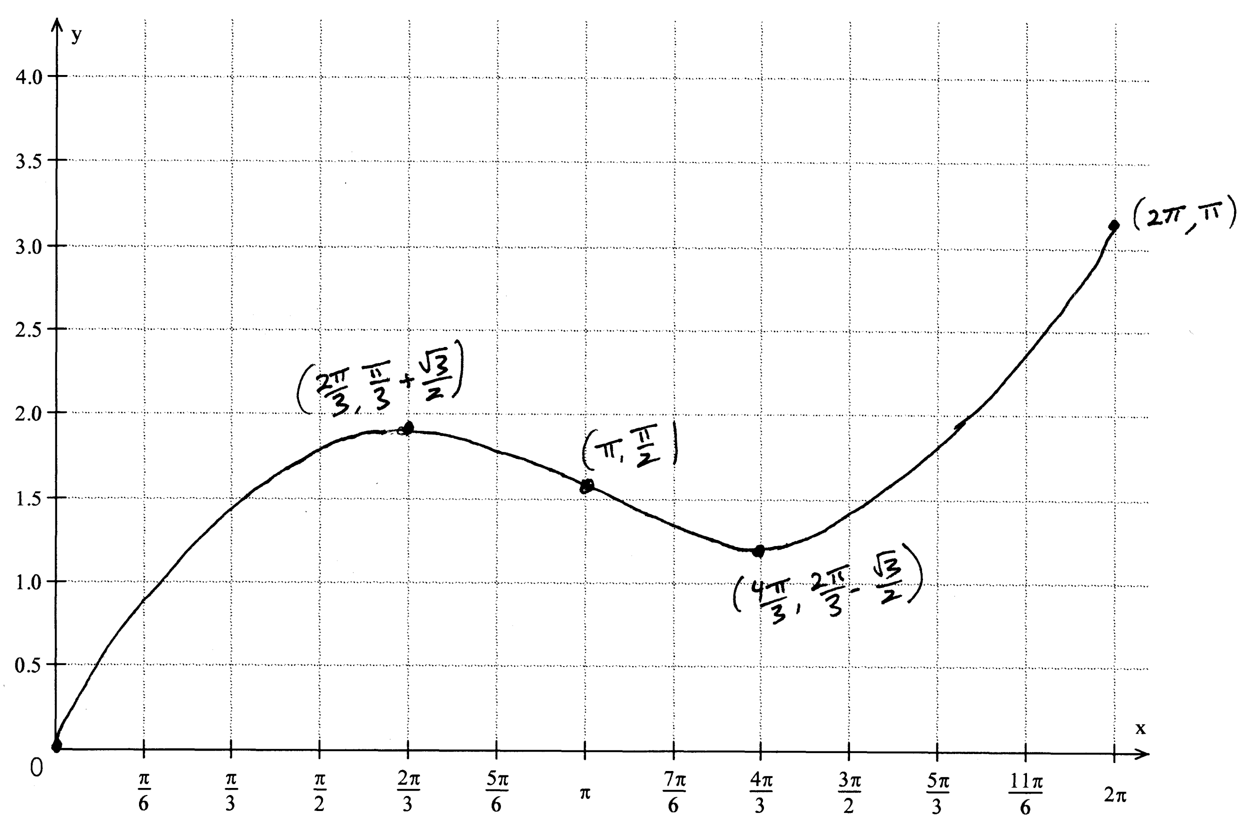
[3]

Intervals	$(0, \pi)$	$(\pi, 2\pi)$
f''	−	+
f	∩	∪

P.O.I. at $(\pi, \frac{\pi}{2})$

(c) Graph the function and label **all** critical points and inflection points.

[3]



4. Use **differentials** to approximate $\sqrt[3]{999}$. Express your answer as a fraction reduced to lowest terms.

$$f(x) = \sqrt[3]{x} = x^{1/3}$$

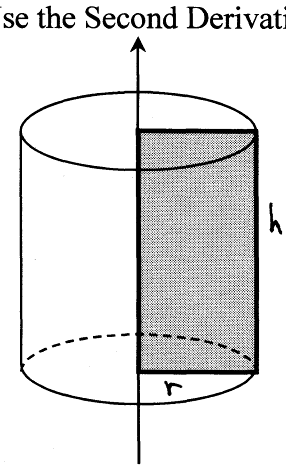
$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

$$f(x+\Delta x) \approx f(x) + f'(x)\,dx = x^{1/3} + \frac{1}{3x^{2/3}}\,dx$$

[3] $$\sqrt[3]{999} = \sqrt[3]{1000 + (-1)} \approx 1000^{1/3} + \frac{1}{3(1000)^{2/3}}(-1)$$

$$= 10 - \frac{1}{300} = \frac{2999}{300}$$

5. Consider a rectangle of perimeter 12 inches. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume? Use the Second Derivative Test to verify that your answer produces a maximum.



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Maximize $V = \pi r^2 h$

Subject to $2r + 2h = 12$

$r + h = 6$

$h = 6 - r$

$$V = \pi r^2 (6-r) = \pi(6r^2 - r^3), \quad 0 \le r \le 6$$

[5] $$V' = \pi(12r - 3r^2) = 3\pi r(4 - r)$$

$V' = 0$ at $r = 0$ and $\underbrace{r = 4}_{\text{critical value}}$

$$V'' = \pi(12 - 6r)$$

$V''(4) = -12\pi < 0$ $\therefore$ max at $r = 4$ by SDT.

When $r = 4$, $h = 2$

$\therefore$ Rectangle should be 4in x 2in with revolution about the shorter edge.