CAMOSUN

**Mathematics 100 Test #2A** **Name:** SOLUTIONS

**Instructor: George Ballinger** **Section:** ____________

**Term: Fall, 2018**

The Sharp EL-531 calculator may be used on this test.
Show all of your work in the space provided.
The number of marks for each question is indicated in brackets.

**Mark:** ____ / 25

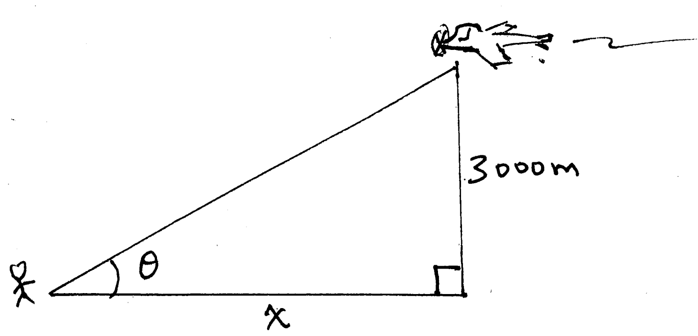
1. If Newton's Method were used to approximate a zero of $f(x) = x - 2\sqrt{x+1}$ using an initial approximation of $x_1 = 3$, then compute the next approximation $x_2$.

$$f'(x) = 1 - \frac{1}{\sqrt{x+1}}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{-1}{\left(\frac{1}{2}\right)} = 5$$

[3]

2. An observer on the ground sights an approaching plane flying at a constant speed at an altitude of 3,000 m. From his point of view, the plane's angle of elevation is increasing at a rate of 0.01 rad/sec when the angle is $\pi/6$. What is the speed of the plane?




[5]

Given: $\frac{d\theta}{dt} = 0.01$ rad/sec when $\theta = \frac{\pi}{6}$

Find: $\frac{dx}{dt}$

Equation: $\tan\theta = \frac{3000}{x} \Rightarrow x = 3000\cot\theta$

$$\frac{dx}{dt} = -3000\csc^2\theta \, \frac{d\theta}{dt}$$

When $\theta = \pi/6$, $\frac{dx}{dt} = -3000 \csc^2\frac{\pi}{6} \cdot 0.01$

$$= -3000 \cdot 4 \cdot 0.01$$

$$= -120 \text{ m/s}$$

Plane is travelling at a speed of 120 m/s toward observer.

3. Let $f(x) = x + 2\sin x$ for $0 \le x \le 2\pi$.

(a) Find the open intervals on which $f$ is increasing or decreasing and find the coordinates of all critical points. Classify each critical point as a relative maximum, minimum or neither.

$$f'(x) = 1 + 2\cos x = 0 \Rightarrow \cos x = -\tfrac{1}{2} \Rightarrow x = \tfrac{2\pi}{3}, \tfrac{4\pi}{3}$$

$$f\left(\tfrac{2\pi}{3}\right) = \tfrac{2\pi}{3} + \sqrt{3} \approx 3.8; \quad f\left(\tfrac{4\pi}{3}\right) = \tfrac{4\pi}{3} - \sqrt{3} \approx 2.5$$

[3]

| Intervals | $(0, \frac{2\pi}{3})$ | $(\frac{2\pi}{3}, \frac{4\pi}{3})$ | $(\frac{4\pi}{3}, 2\pi)$ |
|---|---|---|---|
| $f'$ | + | — | + |
| $f$ | ↗ | ↘ | ↗ |

By FDT, Local max at $\left(\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3}\right)$ and local min at $\left(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3}\right)$

(b) Find the open intervals on which the graph of $f$ is concave upward or concave downward and find the coordinates of all inflection points.

$$f''(x) = -2\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$$

$$f(0) = 0, \; f(\pi) = \pi \approx 3.1; \; f(2\pi) = 2\pi \approx 6.3$$

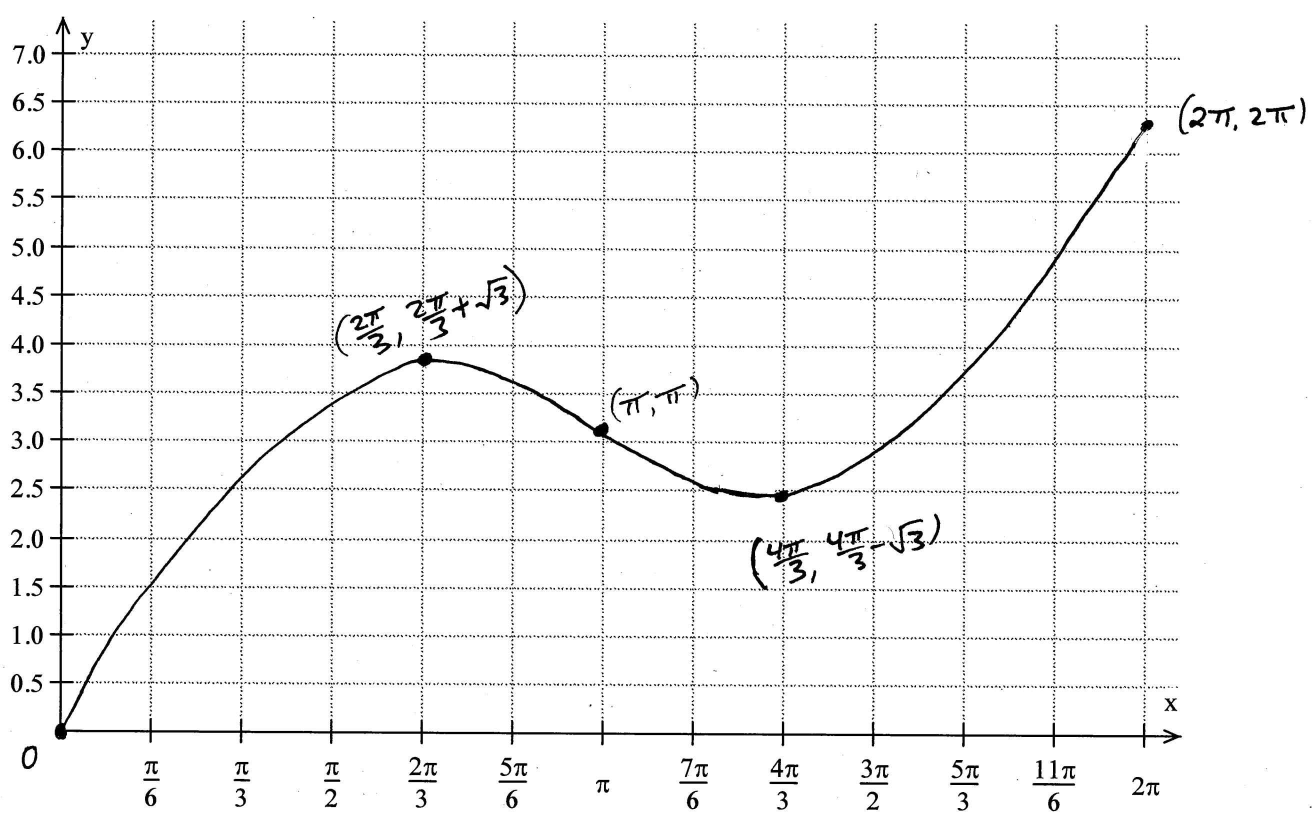
[3]

| Intervals | $(0, \pi)$ | $(\pi, 2\pi)$ |
|---|---|---|
| $f''$ | − | + |
| $f$ | ∩ | ∪ |

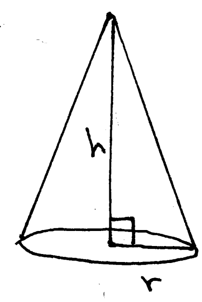
P.O.I. at $(\pi, \pi)$

(c) Graph the function and label **all** critical points and inflection points.

[3]



4. A stalagmite has the shape of a circular cone with height 200 mm and base radius 40 mm. If after a century its height increases by 3 mm and its base radius decreases by 0.5 mm, then using **differentials** approximate its change in volume. Round your approximation to the nearest $mm^3$.




[3]

$V = \frac{1}{3}\pi r^2 h$

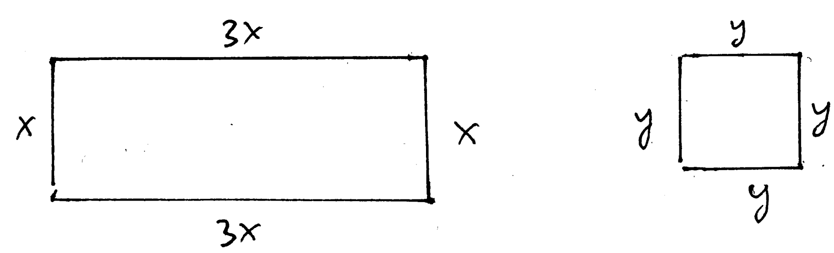
$\Delta V \approx dV = \frac{\pi}{3}\left(r^2\,dh + 2r\,dr\,h\right)$

$= \frac{\pi}{3}\left(40^2(3) + 2(40)(-0.5)(200)\right)$

$= -\frac{3200\pi}{3} \approx -3351\ mm^3$

$\therefore$ Volume decreases by approx. $3351\ mm^3$.

5. You have 140 cm of wire, and you have to use part of this wire to make a rectangle that is three times as long as it is wide, and the rest of the wire (if there is any left) to make a square. What should the dimensions of the shapes be if you want the total area to be as small as possible? Use the Second Derivative Test to verify that your answer produces a minimum.




[5]

Minimize $A = 3x^2 + y^2$

subject to $8x + 4y = 140$

$2x + y = 35$

$y = 35 - 2x$

$A = 3x^2 + (35-2x)^2, \quad 0 \le x \le \frac{35}{2}$

$= 3x^2 + 1225 - 140x + 4x^2$

$= 7x^2 - 140x + 1225$

$A' = 14x - 140 = 0 \Rightarrow x = 10$ (critical value)

$A'' = 14 > 0 \quad \therefore$ min at $x = 10$ by SDT.

When $x = 10$, $y = 15$

$\therefore$ Rectangle should measure 30cm × 10cm and Square should measure 15cm × 15cm.