CAMOSUN

Name: SOLUTIONS

Mark: $\overline{}$ 25

MATH 100 (Winter, 2026)

Test 1

1. (2 marks) Find the equation of the line, in general form, that passes through the point $(-3, 2)$ and is perpendicular to the line $x + y = 7$.

$\underbrace{x+y=7}$

$y = -x + 7$ has slope -1

$\therefore$ $\perp$ line has slope 1

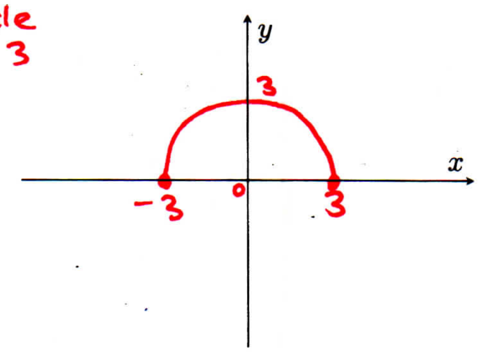
$y - 2 = 1(x+3) \Rightarrow x - y = -5$

2. (2 marks) Sketch the graph of the function $f(x) = \sqrt{9 - x^2}$ and state its domain and range using interval notation.

Semicircle radius 3

Domain = $[-3, 3]$

Range = $[0, 3]$



3. (2 marks) Solve the equation $\cos^2\theta + \sin\theta = 1$ for θ, where $0 \le \theta \le 2\pi$.

$1 - \sin^2\theta + \sin\theta = 1$

$\sin^2\theta - \sin\theta = 0$

$\sin\theta(\sin\theta - 1) = 0$

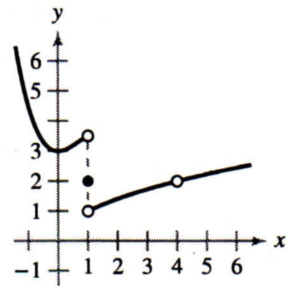
$\sin\theta = 0$ or $\sin\theta = 1$

$\theta = 0, \pi, 2\pi$ $\quad$ $\theta = \frac{\pi}{2}$

4. (1 mark) Given the graph of the function $f(x)$, evaluate the following limits. Answer "dne" if the limit does not exist.

$\lim_{x\to 1} f(x) =$ d.n.e.

$\lim_{x\to 4} f(x) =$ 2



5. Evaluate the limits. If they do not exist, then determine whether they are ∞, $-\infty$ or neither.

(a) (2 marks) $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$

$$= \lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3} = \lim_{x\to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)}$$

$$= \lim_{x\to 4} \frac{x-4}{(x-4)(\sqrt{x+5}+3)} = \lim_{x\to 4} \frac{1}{\sqrt{x+5}+3} = \frac{1}{6}$$

(b) (1 mark) $\lim_{x\to(1/2)^-} x \sec \pi x = \lim_{x\to(\frac{1}{2})^-} \frac{x}{\cos \pi x} = \frac{1/2}{\cos((\frac{\pi}{2})^-)} = \frac{1/2}{0^+} = \infty$

(d.n.e.)

6. (3 marks) Verify that the Intermediate Value Theorem applies to $f(x) = x^2 + x - 1$ on the interval $[0, 5]$ for $k = 11$, and then find the value of c in the interval satisfying $f(c) = k$ guaranteed by the theorem.

$$\left.\begin{array}{l} f(0) = -1 < 11 \\ f(5) = 29 > 11 \end{array}\right\} \quad f(0) < 11 < f(5)$$

and f is continuous on $[0, 5]$. $\therefore$ IVT applies

$f(x) = 11 \Rightarrow x^2 + x - 1 = 11 \Rightarrow x^2 + x - 12 = 0 \Rightarrow (x+4)(x-3) = 0$

$\Rightarrow \underbrace{x = -4}_{\text{discard (not in } [0,5])}$ or $x = 3 \rightarrow \therefore c = 3$

7. (3 marks) Use the **limit definition** of a derivative to find $f'(x)$, where $f(x) = \dfrac{1}{x-1}$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x - 1} - \frac{1}{x-1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x-1) - (x+\Delta x - 1)}{(x+\Delta x - 1)(x-1)\,\Delta x} = \lim_{\Delta x \to 0} \frac{-\Delta x}{(x+\Delta x-1)(x-1)\,\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x - 1)(x-1)} = \frac{-1}{(x-1)^2}$$

8. Find and simplify the derivatives of the following functions.

(a) (1 mark) $f(t) = t^2 \sin t$

$$f'(t) = t^2 \cos t + 2t \sin t$$

(b) (3 marks) $y = \dfrac{x}{\sqrt{x^2+1}}$.

$$y' = \frac{\sqrt{x^2+1} \cdot 1 - x \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x}{\left(\sqrt{x^2+1}\right)^2} = \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

$$= \frac{x^2+1-x^2}{(x^2+1)^{3/2}} = \frac{1}{(x^2+1)^{3/2}}$$

9. (3 marks) Find the equation of the tangent line to the graph of $f(x) = \tan^2 x$ at the point $\left(\frac{\pi}{4}, 1\right)$. Write your answer in slope-intercept form.

$f'(x) = 2\tan x \sec^2 x$

$f'(\frac{\pi}{4}) = 2(1)(\sqrt{2})^2 = 4$ (slope)

$y - 1 = 4(x - \frac{\pi}{4})$

$y = 4x - \pi + 1$

10. (2 marks) Prove $\dfrac{d}{dx}[\cot x] = -\csc^2 x$.

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$$

$$= -\frac{(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x.$$