

Summation

Sigma Notation

$$\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \cdots + a_n,$$

where i is the **index of summation**, which often starts at 1 but may start at any integer less than or equal to n . Other letters, particularly j and k , may be used in place of i . Here a_i represents the i^{th} **term** of the sum.

Summation Properties

1. $\sum_{i=1}^n k a_i = k \sum_{i=1}^n a_i$
2. $\sum_{i=1}^n (a_i \pm b_i) = \sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i$

Summation Formulas

1. $\sum_{i=1}^n c = c + c + c + \cdots + c = cn$
2. $\sum_{i=1}^n i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
3. $\sum_{i=1}^n i^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
4. $\sum_{i=1}^n i^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$