Guidelines for Solving Optimization Problems

1. After carefully reading the problem, make a sketch and introduce variables for quantities that are both given and to be determined.

2. Decide on the variable that is to be maximized or minimized and rephrase the problem in the format:
   Maximize or Minimize _____
   Subject to the constraint _____

3. Express the quantity being maximized or minimized in terms of just one variable by using the constraint to eliminate other variables.

4. Find the domain of the continuous function you are trying to optimize.

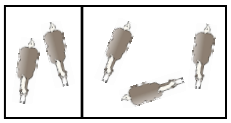
5. Find critical numbers of your function that belong to the domain.

6. Depending on the interval and number of critical values, do the following:
   (a) <u>If the domain is a closed interval.</u> Evaluate the function at the endpoints of the interval as well as at critical numbers to identify the absolute maximum or absolute minimum.
   (b) <u>If the domain is an open interval containing only one critical number.</u> Use the First or Second Derivative Test to determine whether the function has a local extremum at the critical number. If it does, then the local extremum will be an absolute extremum.
   (c) <u>If the domain is an open interval containing more than one critical number.</u> Evaluate the function at the critical numbers as well as find the limits at the endpoints of the interval to determine whether an absolute maximum or minimum occurs at one of the critical points.

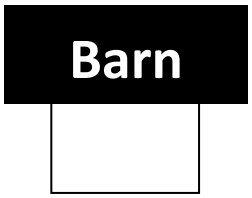
7. Reread the problem and check that you have answered the question that was asked. Don't forget appropriate units.

Optimization Word Problems

1. A rancher has 200 meters of fencing with which to enclose two adjacent corrals, one smaller and one larger as shown in the picture. What should the outer dimensions be so that the enclosed area is a maximum?

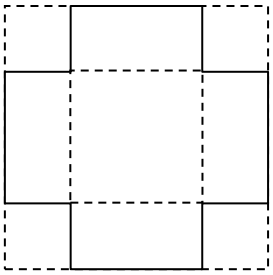


2. A farmer has a barn that is 50 feet long. He has 200 ft of fence with which to enclose a rectangular garden one of whose sides is to be along the barn. What are the dimensions of the largest garden that can be enclosed?

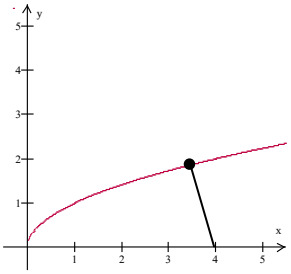


3. Find two positive numbers such that the product is 192 and the sum of the first and three times the second is a minimum.

4. An open box is to be made from a square piece of material, 24 cm on a side, by cutting out equal squares from each corner and turning up the sides. What will be the volume of the largest such box?



5. Find the point on the graph of $f(x) = \sqrt{x}$ that is closest to the point (4, 0).



6. Walmart wants to design a cylindrical can for their Great Value brand of corn niblets that uses the least amount of metal but holds 341 mL of corn. What should the height and radius of the can be?
Note that 1 cm^3 = 1 mL.

7. An open box with a square base is to have a volume of 12,000 cm^3. The material for the bottom of the box costs \$6/cm^2 while the material for the sides of the box costs only \$2/cm^2. What should the dimensions of the box be to minimize the total cost?