Math 100 Review Questions for the Final Exam

These questions together with your in-class tests and the additional chapter review questions on this website will be a good review for the final exam.

1a) $\lim_{x->0} \frac{7x - \sin 5x}{x}$ b) $\lim_{x \to 0} \frac{7x}{\sqrt{x+4} - 2}$

2a) $\lim_{x \to -1^-} \frac{3}{x+1}$ b) $\lim_{x \to -1^-} \frac{3x^2 + 6x + 3}{x+1}$ c) $\lim_{x \to -1^-} \frac{3|x+1|}{x+1}$

3. Let $f(x) = \begin{cases} \cos x & -2p \le x < 0 \\ 1 & x = 0 \\ x^2 - 1 & 0 < x \le 2 \end{cases}$

a) $\lim_{x \to 0^+} f(x)$ b) $\lim_{x \to 0^-} f(x)$ c) $\lim_{x \to 0} f(x)$ d) $f(0)$

4. Consider $f(x) = \begin{cases} \frac{1}{x} & x \ne 0 \\ 3 & x = 0 \end{cases}$.

a) $\lim_{x \to 0^+} f(x)$ b) $f(0)$ c) Classify the discontinuity at $x = 0$.

5. $\lim_{x \to \infty} \frac{6}{1 + e^{-2x}}$ 6. $\lim_{x \to 5^+} \ln(x - 5)$

7. Suppose $h(t) = \frac{2t^3}{8 + t^3}$

a) $\lim_{t \to \infty} h(t)$

b) What does the answer to a) tell you about the graph of the function?

8a) State the definition of a derivative.

b) Using the definition, find the derivative of $f(t) = 8(3t)^{-2}$. Then check your answer using standard differentiation rules.

9a) $\left.\dfrac{d}{dx}(-e^{-x^2})\right|_{x=1}$ (answer to the nearest tenth)

b) $\dfrac{d}{dx}\left[\ln\left(\dfrac{x^7\sqrt{4x+1}}{7^x\left(x^3+5\right)^3}\right)\right]$ c) $\dfrac{d}{dx}\left(3^x+x^3+3^3+x^x\right)$ d) $\dfrac{d}{dx}\left[(3x)^{\cos x}\right]$

e) $d\left(t\csc 5t\right)$ f) $\dfrac{d}{dq}\left[\sin\left(q\tan q\right)^2\right]$ (Don't simplify the answer)

g) $\dfrac{d}{dx}\displaystyle\int_3^x \sqrt{9+t^6}\,dt$

10. Suppose f and g are differentiable functions with the values given in the following table. Find $h'(2)$ where $h(x)=f(g(x))$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	5	5	6	3
5	2	8	7	4

11. Find the equation of the tangent line to the graph of $x^2y^2=\left(y+1\right)^2\left(9-y^2\right)$ at the point $\left(0,-3\right)$.

12. Suppose $f'(3)=0$ and $f''(3)=7$. What can you conclude about the point $\left(3, f(3)\right)$?

13. Find all points of inflection of $f(x)=2x^6-5x^4$.

14. Suppose $f(x)=\dfrac{x^2-5x+4}{x^2}$. Find

a) the first and second derivatives and write the answers in simplest factored form. (Hint: rewrite before you differentiate.)
b) the domain
c) the intercepts
d) all asymptotes
e) intervals where the function is increasing and decreasing and any relative extrema
f) intervals where the function is concave up and down and any inflection points
g) Using the above information, sketch a graph of the function.

15. Let a be a positive constant. Find the critical numbers for the function $f(x) = xe^{-\alpha x^2}$.

16. A piece of wire 100 cm long is going to be cut into 12 pieces and used to construct the skeleton (edges only) of a rectangular box. The base of the box is to be constructed so that the length is twice the width. The resulting frame is then covered with paper. What are the dimensions of the box with the largest

a) volume?

b) surface area?

17. An aircraft is flying horizontally at a constant altitude of 1 km. At a certain moment, stationary radar on the ground notes that the speed of the aircraft is 300 km/hr and that the angle of elevation is 30^{o} and is decreasing.

a) Is the plane flying toward the radar or away from it?

b) How fast is the angle of elevation decreasing at that moment?

c) How fast is the distance between the radar and aircraft changing at that instant?

18. I am standing 40 metres from a straight road watching a cyclist on the road through my binoculars. At the instant the cyclist is 50 metres away from me, my binoculars are rotating at 0.3 radians per second. How fast is the cyclist moving?

19. At a certain moment, the ladder on a fire truck is 20 m long and makes an angle of $p\ /3$ radians with the horizontal. At what rate is a firefighter on the far end of the ladder rising if

a) the ladder is rotating upwards at a rate of 0.1 radians/s?

b) the ladder is rotating upwards at a rate of 0.1 radians/s and extending in length at a rate of 0.5 m/s?

20. Suppose a particle is moving along the x-axis with acceleration given by $a(t) = \sqrt{t+4}$. If the particle starts from rest at $x = 0$, find the position function $x(t)$.

21. A particle moves along the along the x-axis with velocity $\dfrac{1}{1+t}$ at time t. If it's initial position is at $x = 2$, what is its acceleration when it passes $x = 3$? (exact value)

22 a) $\int \left(1-2\sqrt{x}\right)^2 dx$

b) $\int \sqrt{1+\cos 2x}\ \sin 2x\, dx$

c) $\int \left(x-e^{3x}\right)dx$

d) $\int_1^e \frac{\sin(\ln x)}{x}dx$ (exact value)

e) $\int \frac{1}{1-\sin x}dx$

f) $\int \frac{\cos x}{1-\sin x}dx$

g) $\int \frac{x^3}{x^4+1}dx$

h) $\int \frac{x^4+1}{x+1}dx$

i) $\int \sec 5x\,dx$

23. Find and sketch the area bounded by the curve $y = x(x-2)^4$ and the x-axis. (Hint: You don't need a great picture; you only need to know the x-intercepts and whether the graph lies above or below the x – axis.)

24a) Give the definition of $\int_a^b f(x)dx$.

b) Using the definition, find the area under the curve $f(x) = 3x^2 + 1$ from x = 0 to x = 2.

25. Suppose you wanted to find the area under the curve $y = f(x)$ on the interval $[-6,20]$ using the definition. Assuming that the n rectangles all have the same width, what is the x-coordinate at the right endpoint of the i'th rectangle?

26. Find $y = f(x)$ given $f''(x) = x^2 + \cos x$, $f'(0) = 7$ and $f(0) = 2$.

27. Find the general solution of the differential equation $\left(x^2+4\right)\frac{dy}{dx} - xy = 0$.

28. Water is poured into a glass at a rate of t^2 ml/s. If you initially have 10 ml of water in your glass, how much water will be in the glass at $t = 6$ seconds?

29. Suppose that the rate of decay of a radioactive substance is proportional to the amount of the radioactive material present.

a) Express the above as a differential equation and using the technique of separation of variables, solve your DE.

b) After 3 days, a sample of radon-222 decayed to 58% of its original amount. How much of the original amount will be left after 11 days?

30. Let $P = f(t)$ be the population (in thousands) in a town in year t (where $t = 0$ corresponds to 1970). Interpret $f(20)$ and $f^{-1}(20)$.

31. Use the fact that the cube root of 27 is 3 and either linear approximation or differentials to estimate the cube root of 20.

32. Use Newton's Method to find the point of intersection of the two functions $y = \sin x$ and $y = x/2$. Just do one iteration and use 1.4 for your first estimate.

33. Describe the purpose of Newton's method and explain how it works with the aid of a picture. (You don't need to refer to its formula at all – just describe the procedure geometrically.)

34. Estimate $\int_0^1 \sin x^2 \, dx$ the following using both the Trapezoidal Rule and Simpson's Rule with $n = 4$.