

Math 100 Final Exam Review Questions

1. Evaluate the following limits:

a) $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$ b) $\lim_{x \rightarrow 0} \frac{1}{x+3} - \frac{1}{3}$ c) $\lim_{x \rightarrow 0} \frac{\sin x}{x^2 + 2x}$ d) $\lim_{x \rightarrow 2^+} \frac{x+1}{x-2}$

2. Sketch a graph which satisfies all of the following conditions:

$$\lim_{x \rightarrow 3^-} f(x) = 0 \quad \lim_{x \rightarrow 3^+} f(x) = -2 \quad f(3) = 1$$

3. Use the definition of the derivative to find $f'(x)$ for the following functions:

a) $f(x) = \sqrt{x-3}$ b) $f(x) = \frac{x}{x-1}$

4. Use the rules of derivatives to find y' for each of the following functions:

a) $y = \frac{1}{\sqrt[3]{3-x^3}}$ b) $y = x^3 \sqrt{x+1}$ c) $y = \sqrt{\sin 2x}$

d) $y = \tan^3(4x^2)$ e) $y = \sin x(\sin x + \cos x)$ f) $y = \frac{2x}{1-3x^2}$

5. Find the second derivative of $y = \frac{x+2}{x-3}$.

6. Find the equation of the tangent line to the graph of $y = \tan x$ at the point $\left(\frac{\pi}{4}, 1\right)$.

7. Use implicit differentiation to find y' for the following:

a) $2x^2 + xy + 3y^2 = 0$ b) $x\sqrt{y} + y^2 = x$

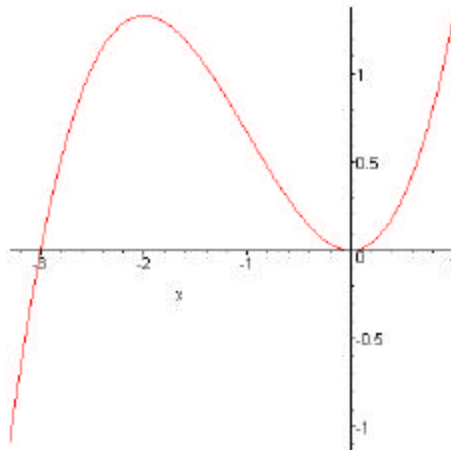
8. Find the slope of the tangent line to the graph of $x^4 + 4x^2y^3 + y^2 = 2y$ at the point $(1, -1)$.

9. Find the equation of the tangent line to $x^2 + 2y^2 = 3$ at the point $(1, 1)$.

10. A balloon rises directly up at a rate of 8 ft/sec from a point 60 feet from an observer. Find the rate of change of the angle of elevation when the balloon is 25 feet above the ground.

11. A 5 meter long ladder is leaning against a wall. The bottom of the ladder is moving away from the wall at a rate of 0.4 m/sec. How fast is the top of the ladder descending when the foot of the ladder is 3 meters from the wall.

12. Find all critical numbers of $y = x\sqrt{2x+1}$.
13. Find the absolute extrema for $f(x) = \sin x + \cos x$ on the interval $[0, 2\pi]$.
14. Find the value of c predicted by the Mean Value Theorem for $f(x) = -\frac{1}{x}$ on the interval $[-3, -\frac{1}{2}]$.
15. Why does the Mean Value Theorem not apply to $f(x) = \frac{2}{(x+1)^2}$ on the interval $[-3, 0]$.
16. Find the intervals on which $f(x) = -x^3 + 9x^2 - 24x + 18$ is increasing and decreasing.
17. Find all relative extrema for $f(x) = x^4 - 4x^3 + 4x^2 + 1$.
18. Find all inflection points of $y = 2x(x-4)^3$.
19. Sketch the graph of $f'(x)$ if the graph of $f(x)$ is as follows:



20. Find all horizontal asymptotes of :

a) $y = \frac{3-2x^2}{5x^3-2x^2}$

b) $y = \frac{5x}{\sqrt{4x^2+3}}$

21. Evaluate $\lim_{x \rightarrow \infty} \frac{5-3x^3}{3x^3+2x+1}$.

22. Find all critical points and inflection points and sketch the graphs of the following functions:

a) $f(x) = x^3 - 3x + 2$ b) $g(x) = 2\sin x + \sin 2x$ on $[0, 2\pi]$ c) $h(x) = \frac{x^2}{x-1}$

23. A rectangular lot is to contain 3750 square meters. If the lot is to be divided down the middle into two rectangular enclosures, then what dimensions will minimize the amount of fencing needed and how much fencing will be used ?

24. An open box is made by cutting squares out of the corners of a 3 foot by 7 foot piece of cardboard and folding up the sides. What size squares should be removed to create a box with maximum volume and what is the maximum volume ?

25. What point on the curve $y = \sqrt{x+1}$ is closest to the point $(3, 0)$?

26. Use Newton's Method to solve $x^3 - 2x = 2$. Use an initial guess of $x_1 = 1.5$ and give an answer which is accurate to 4 decimal places.

27. Use differentials to find the approximate amount the volume of a cube with side length 3 cm will change if the side length is increased by 0.01 cm.

28. Use differentials to approximate: a) $\sqrt{3.9}$ b) $\sqrt[3]{64.2}$.

29. Evaluate the following indefinite integrals:

a) $\int \frac{x^4 - x^3}{x^2} dx$ b) $\int \frac{\sin^3 q}{1 - \cos^2 q} dq$ c) $\int 2 \csc^2 3x dx$ d) $\int \sec 3x \tan 3x dx$

30. Use the limit process to find the area under the following curves and above the x - axis :

a) $f(x) = -x^2 + 4$ on the interval $[0, 2]$ b) $f(x) = 2x - x^2$ on the interval $[0, 2]$

31. Evaluate the following definite integrals:

a) $\int_{-1}^1 \sqrt[3]{t} - 2 dt$ b) $\int_{\frac{p}{4}}^{\frac{p}{2}} \sec^2 x dx$

32. Find the average value of the function $f(x) = \sin x$ on the interval $[\frac{p}{4}, \frac{p}{2}]$.

33. Find the value of c guaranteed by the Mean Value Theorem for Integrals for the function $f(x) = 3x^2 - 2$ on the interval $[0, 2]$.

34. Evaluate the following:

a) $\int x\sqrt{4-9x^2} dx$ b) $\int_0^1 x\sqrt{x^2+1} dx$ c) $\int \sin^3 3x \cos 3x dx$ d) $\int \frac{\sec^2 2x}{\sqrt{\tan 2x}} dx$

35. Use the Trapezoidal Rule with $n = 4$ intervals to approximate the following:

a) $\int_2^3 \frac{1}{(x-1)^2} dx$

b) $\int_0^p \sin x dx$

36. Find y' for the following:

a) $y = \ln \sqrt{x^2 + 4}$ b) $y = x^3 \ln(2x + 1)$ c) $\ln(xy) = x + y$ d) $y = \frac{x^3 \sqrt{2x + 3}}{(x - 2)^2}$

37. Evaluate the following:

a) $\int \frac{2x+1}{x+1} dx$ b) $\int \frac{\ln \sqrt{x}}{x} dx$ c) $\int \sec 4x dx$ d) $\int x \cot(x^2) dx$

38. Find $f^{-1}(x)$ if $f(x) = 3x^3 - 1$.

39. If $f(x) = 2x^2 + 1$, $x \geq 0$ then find $(f^{-1})'(9)$.

40. Find y' for the following:

a) $y = e^{-\frac{3}{x}}$ b) $y = e^{\sin \sqrt{x}}$ c) $y = \ln \left(\frac{e^x}{e^x + 1} \right)$ d) $y = \frac{2}{2x + e^{2x}}$

41. Evaluate the following:

a) $\int \frac{1}{x^2 e^{\frac{2}{x}}} dx$ b) $\int \frac{e^x}{\sqrt{e^x + 1}} dx$ c) $\int_1^4 e^{2x} dx$ d) $\int_{-2}^1 2^{-x} dx$

42. Find y' for the following:

a) $y = x^{1-x}$ b) $y = x^{e^x}$ c) $y = x^3 3^x$

43. Solve the following differential equations:

a) $\frac{dy}{dx} = \frac{3x^3}{1+x^2}$ b) $(4-x) \frac{dy}{dx} = -2y$ c) $\frac{dy}{dx} = \frac{\sin x}{\cos y}$ d) $\frac{dy}{dx} = 500 - y$, $y(0) = 7$

44. A bacteria population grows exponentially so that an initial population of 500 bacteria grows to 1000 in two hours. How long will it take until 2500 bacteria are present?