

# Properties of Limits

1. **Basic Limits:** Let  $b$  and  $c$  be real numbers and let  $n$  be a positive integer.

(a)  $\lim_{x \rightarrow c} b = b$

(b)  $\lim_{x \rightarrow c} x = c$

(c)  $\lim_{x \rightarrow c} x^n = c^n$

2. **Properties of Limits:** Let  $b$  and  $c$  be real numbers, let  $n$  be a positive integer, and let  $f$  and  $g$  be functions with the following limits.

$$\lim_{x \rightarrow c} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = K$$

(a) Scalar multiple:  $\lim_{x \rightarrow c} [bf(x)] = bL$

(b) Sum or difference:  $\lim_{x \rightarrow c} [f(x) \pm g(x)] = L \pm K$

(c) Product:  $\lim_{x \rightarrow c} [f(x)g(x)] = LK$

(d) Quotient:  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{K}$  (provided  $K \neq 0$ )

(e) Power:  $\lim_{x \rightarrow c} [f(x)]^n = L^n$

3. **Limit of a Composite Function:** If  $f$  and  $g$  are functions such that  $\lim_{x \rightarrow c} g(x) = L$  and  $\lim_{x \rightarrow L} f(x) = f(L)$ , then

$$\lim_{x \rightarrow c} f(g(x)) = f\left(\lim_{x \rightarrow c} g(x)\right) = f(L).$$

4. **Squeeze Theorem:** If  $h(x) \leq f(x) \leq g(x)$  for all  $x$  in an open interval containing  $c$ , except possibly at  $c$  itself, and if

$$\lim_{x \rightarrow c} h(x) = L = \lim_{x \rightarrow c} g(x),$$

then  $\lim_{x \rightarrow c} f(x)$  exists and equals  $L$ .