Interest

Suppose an initial principal amount P is invested at an annual interest rate r. Then after t years the amount (or balance) A of the investment will be

1. $A = P(1+rt)$, if interest is calculated only on the principal amount (i.e. simple interest)
2. $A = P\left(1+\frac{r}{n}\right)^{nt}$, if interest is compounded n times per year
   (a) $A = P(1+r)^t$, if interest is compounded annually ($n=1$)
   (b) $A = P\left(1+\frac{r}{2}\right)^{2t}$, if interest is compounded semi-annually ($n=2$)
   (c) $A = P\left(1+\frac{r}{4}\right)^{4t}$, if interest is compounded quarterly ($n=4$)
   (d) $A = P\left(1+\frac{r}{12}\right)^{12t}$, if interest is compounded monthly ($n=12$)
   (e) $A = P\left(1+\frac{r}{365}\right)^{365t}$, if interest is compounded daily ($n=365$)
3. $A = Pe^{rt}$, if interest is compounded continuously

Example: Suppose $1,000 is invested at a rate of 6% for three years, i.e. $P=1000$, $r=0.06$ and $t=3$. Then after three years the balance A will be

$A = 1000(1+(0.06)(3)) = \$1{,}180.00$,	if interest is simple
$A = 1000(1+0.06)^3 = \$1{,}191.02$,	if interest is compounded annually (6% per year)
$A = 1000(1+0.03)^6 = \$1{,}194.05$,	if interest is compounded semi-annually (3% per half-year)
$A = 1000(1+0.015)^{12} = \$1{,}195.62$,	if interest is compounded quarterly (1.5% per quarter)
$A = 1000(1+0.005)^{36} = \$1{,}196.68$,	if interest is compounded monthly (0.5% per month)
$A = 1000(1+0.000164384)^{1095} = \$1{,}197.20$,	if interest is compounded daily (0.0164384% per day)
$A = 1000e^{(0.06)(3)} = \$1{,}197.22$,	if interest is compounded continuously

In the case of continuously compounded interest, the rate at which the investment grows is directly proportional to the value of the investment.

Note that $P\left(1+\frac{r}{n}\right)^{nt} \to Pe^{rt}$ as $n \to \infty$. To see why, observe that

$$\lim_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt} = P\lim_{n\to\infty}\left(1+\frac{1}{n/r}\right)^{(n/r)rt} = P\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{xrt} = P\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]^{rt} = Pe^{rt},$$

where we replaced n/r by x and observed that $x \to \infty$ as $n \to \infty$.