Properties of Infinite Limits

Let c and L be real numbers and let f and g be functions.

1. If $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} g(x) = \infty$, then $\lim_{x\to c}(f(x) + g(x)) = \infty$.
2. If $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} g(x) = L$, then $\lim_{x\to c}(f(x) + g(x)) = \infty$.
3. If $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} g(x) = L$, then $\lim_{x\to c}(f(x) - g(x)) = \infty$.
4. If $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} g(x) = \infty$, then $\lim_{x\to c}(f(x)g(x)) = \infty$.
5. If $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} g(x) = L$, then
   (a) $\lim_{x\to c}(f(x)g(x)) = \infty$, if $L > 0$.
   (b) $\lim_{x\to c}(f(x)g(x)) = -\infty$, if $L < 0$.
6. If $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} g(x) = L$, then $\lim_{x\to c} \frac{g(x)}{f(x)} = 0$.

Note: Similar properties hold for one-sided limits, where $x \to c^+$ or $x \to c^-$, and for limits where the functions approach $-\infty$ instead of ∞ as x approaches c.

Note: These properties can be summarized and abbreviated as follows:

$$\infty + \infty = \infty, \qquad \infty \pm L = \infty, \qquad \infty \cdot \infty = \infty$$
$$\infty \cdot L = \infty \text{ if } L > 0, \qquad \infty \cdot L = -\infty \text{ if } L < 0, \qquad \frac{L}{\infty} = 0$$

Note: Limits of the form $\infty - \infty$, $\infty \cdot 0$, ∞/∞, and $0/0$ are said to be *indeterminate*. They may or may not exist, depending on the nature of the functions f and g.