Increasing and Decreasing Functions

Definition of Increasing and Decreasing Functions

Let f be a function defined on an interval I.

1. f is **increasing** on I if for any two numbers x_1 and x_2 in I, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.
2. f is **decreasing** on I if for any two numbers x_1 and x_2 in I, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

Test for Increasing and Decreasing Functions

Let f be a function that is continuous on $[a, b]$ and differentiable on (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

First Derivative Test

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c, then f has a relative minimum at $(c, f(c))$.
2. If $f'(x)$ changes from positive to negative at c, then f has a relative maximum at $(c, f(c))$.
3. If $f'(x)$ is positive on both sides of c or negative on both sides of c, then $f(c)$ is neither a relative minimum nor a relative maximum.