Fundamental Theorem of Calculus

Let f be a continuous function on $[a, b]$ and suppose F is an antiderivative of f, in other words $F'(x) = f(x)$.

Recall that the **indefinite integral** of f with respect to x is defined by

$$\int f(x)\,dx = F(x) + C,$$

whereas the **definite integral** of f from a to b is defined by

$$\int_a^b f(x)\,dx = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i = \lim_{n \to \infty} \sum_{i=1}^{n} f(a + i\Delta x)\Delta x,$$

where $\Delta x = \dfrac{b-a}{n}$.

Fundamental Theorem of Calculus (FTC)

If f is continuous on $[a, b]$ and F is an antiderivative of f, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Mean Value Theorem for Integrals

If f is continuous on $[a, b]$, then there exists some c in $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a), \text{ or equivalently, } f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Definition of the Average Value of a Function

If f is integrable on $[a, b]$, then the **average value** of f on $[a, b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then for every x in I,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x).$$