Concavity and Second Derivative Test

Definition of Concavity

Let f be a differentiable function defined on an open interval I.

1. The graph of f is **concave upward** on I if f' is increasing on I.
2. The graph of f is **concave downward** on I if f' is decreasing on I.

Test for Concavity

Let f be a function whose second derivative exists on an open interval I.

1. If $f''(x) > 0$ for all x in I, then f is concave upward on I.
2. If $f''(x) < 0$ for all x in I, then f is concave downward on I.

Definition of Point of Inflection

Let f be continuous on an open interval I containing c. The point $(c, f(c))$ is called a **point of inflection** of the graph of f if the graph has a tangent line at $(c, f(c))$ and the concavity changes from upward to downward or from downward to upward at the point.

Note: The tangent line could be vertical, in which case $f'(c)$ is undefined, or nonvertical, in which case $f'(c)$ is defined and represents its slope.

Note: At a point of inflection $(c, f(c))$, $f''(c)$ cannot be positive or negative. Therefore either $f''(c) = 0$ or $f''(c)$ is undefined.

Second Derivative Test

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.

Note: If $f''(c) = 0$, then the test is inconclusive (i.e. it fails). In this case f may have a relative minimum, a relative maximum, or neither. The First Derivative Test should instead be used.