Name: _____

Mark: $\overline{}$ **25**

MATH 100

Assignment 6

1. (2 marks) Differentiate $f(x) = \ln(\sqrt{25 + 4x^2} + 2x)$.

2. (2 marks) Use logarithmic differentiation to find dy/dx, where

$$y = \sqrt{\frac{x^2 + 4}{x^2 - 4}}, \quad \text{for } x > 2.$$

3. (2 marks) Find the average value of $f(x) = \dfrac{\tan^2 x + \sec^2 x}{\tan x}$ on the interval $[\pi/6, \pi/3]$. Simplify your answer as much as possible.

4. (2 marks) Evaluate the definite integral $\displaystyle\int_0^4 \frac{x}{(2x+1)^2}\,dx$.

5. Find the indefinite integrals.

   (a) (2 marks) $\displaystyle\int \frac{1}{x\sqrt{\ln\sqrt{x}}}\,dx$

   (b) (2 marks) $\displaystyle\int \frac{x^3 - 3x^2 - 7x + 18}{x^2 - 6}\,dx$

6. (2 marks) Let $f(x) = x^5 + x^3 + 2x + 1$. Prove that f has an inverse. Then find $(f^{-1})'(-3)$.

7. (2 marks) Differentiate $y = \ln\left(\dfrac{1 - e^x}{1 + e^x}\right)$. Simplify your answer.

8. (2 marks) Find y' if $e^{xy} = x + y$. Express your answer as an algebraic expression, i.e. not involving transcendental functions such as exponential or logarithmic functions.

9. (1 mark) Find the indefinite integral $\displaystyle\int \frac{e^{1/x^4}}{2x^5}\,dx$.

10. (2 marks) Evaluate the definite integral $\displaystyle\int_0^2 \frac{e^{2x+1}}{1-e^{2x+1}}\,dx$. Round your answer to 3 decimal places.

11. (2 marks) Use logarithmic differentiation to find the derivative of $y = (\ln x)^{\cos x}$.

12. (1 mark) Differentiate $y = (20^t - 20^{-t})^{20}$.

13. (1 mark) Find the area of the region bounded by the graphs of $y = 2^{-x}$, $y = 0$, $x = -1$ and $x = 1$.