Name: _____

Mark: $\overline{}$ **25**

MATH 100

Assignment 5

1. (1 mark) Find the indefinite integral.

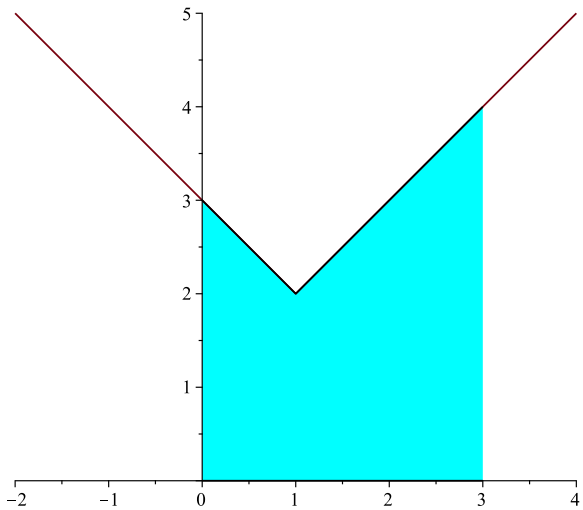
$$\int \csc\theta(\cot\theta - 4\csc\theta)\,d\theta$$

2. (2 marks) Find the particular solution, $f(x)$, of the differential equation

$$f''(x) = 2x^3 - 7x$$

that satisfies the initial conditions $f'(2) = 1$ and $f(1) = 2$.

3. Consider the shaded region.



(a) (1 mark) Approximate the area of the region using 6 rectangles and right endpoints.

(b) (1 mark) Set up a *single* definite integral that represents the area of the shaded region. You do not have to evaluate the integral.

4. (3 marks) Evaluate the definite integral using the limit definition.

$$\int_1^5 (2 - (x-1)^3)\, dx$$

5. Evaluate the definite integrals.

   (a) (2 marks) $\displaystyle\int_{-3}^{-1} \frac{2x^3 + 3x^2 + 1}{x^2}\, dx$

   (b) (2 marks) $\displaystyle\int_{0}^{3} |x^2 - 1|\, dx$

6. (3 marks) Evaluate the definite integral $\displaystyle\int_4^{12} \frac{x}{\sqrt{2x+1}}\,dx$.

7. (2 marks) Find the indefinite integral $\displaystyle\int \frac{\sin 2\theta}{\cos^5 2\theta}\,d\theta$.

8. (3 marks) Find the value(s) of c guaranteed to exist by the Mean Value Theorem for Integrals for the function $f(x) = \sec^2 x$ over the interval $[-\pi/3, \pi/3]$. Round your answer(s) to four decimal places.

9. (2 marks) Suppose f is an odd function and $f(2) = 3$. Evaluate $F(1)$ and $F'(1)$, where

$$F(x) = \int_{-2}^{2x} f(t)\, dt.$$

10. (3 marks) Use the Trapezoidal Rule and Simpson's Rule with $n = 6$ to approximate

$$\int_1^3 \sqrt{x} \sin x \, dx.$$

Round your answers to six decimal places.