MATH 100 **ASSIGNMENT #4** **Name:** SOLUTIONS **Section:**____

25

First work through the recommended practice problems listed in the following table from the 11th edition of *Calculus of a Single Variable* by Larson and Edwards. You do not need to hand these in. Once you have completed these, then do the small sampling of questions below. Write full solutions (not just the final answer) in the space provided.

3.5 Limits at Infinity	13, 15, 19, 21, 23, 27, 29, 31, 33, 35, 41, 43
3.6 A Summary of Curve Sketching	9, 11, 13, 15, 21, 23, 27, 29, 33, 37, 41, 59, 64, 78
3.7 Optimization Problems	5, 7, 11, 13, 15, 17, 19, 21, 25, 29, 33, 35
3.8 Newton's Method	3, 9, 11, 13, 15, 17, 19, 23, 29, 33
3.9 Differentials	7, 9, 13, 15, 19, 21, 23, 25, 27, 33, 35, 37, 39, 41, 43, 45

Sec 3.5 #22: Find the limit. $\lim_{x\to\infty}\frac{5x^3+1}{10x^3-3x^2+7} = \lim_{x\to\infty}\frac{5+\frac{1}{x^3}}{10-\frac{3}{x}+\frac{7}{x^3}} = \frac{5}{10} = \frac{1}{2}$

Sec 3.6 #22: Analyze and sketch a graph of the function $f(x)=\frac{x^3}{x^2-9}$.
Label any intercepts, relative extrema, points of inflection and asymptotes. As part of your analysis check for symmetry and find the intervals on which f is increasing or decreasing and where the graph of f is concave upward or concave downward.

$f(0)=0 \therefore (0,0)$ is y-int

$f(x)=0$ at $x=0 \therefore (0,0)$ is x-int.

$f(x)=\frac{x^3}{(x+3)(x-3)} \quad \therefore x=\pm 3$ are V.A.

$f(-x)=\frac{(-x)^3}{(-x)^2-9} = -\frac{x^3}{x^2-9} = -f(x)$

$\therefore f$ is odd; symmetry about origin

$$\begin{array}{r} x \\ x^2-9\overline{)\,x^3} \\ \underline{x^3-9x} \\ 9x \end{array}$$

$f(x)=x+\frac{9x}{x^2-9}$

$\therefore y=x$ is slant asymptote (no H.A.)

$f'(x)=\frac{(x^2-9)(3x^2)-x^3(2x)}{(x^2-9)^2} = \frac{3x^4-27x^2-2x^4}{(x^2-9)^2}$

$=\frac{x^4-27x^2}{(x^2-9)^2}=\frac{x^2(x^2-27)}{(x^2-9)^2}=0$ at $x=0$ and $x=\pm\sqrt{27}=\pm 3\sqrt{3}$ } critical #'s

Intervals	$(-\infty,-3\sqrt{3})$	$(-3\sqrt{3},-3)$	$(-3,0)$	$(0,3)$	$(3,3\sqrt{3})$	$(3\sqrt{3},\infty)$
f'	+	−	−	−	−	+
f	↗	↘	↘	↘	↘	↗

$\underbrace{\qquad}_{(-3,3)}$

By FDT, local max at $\left(-3\sqrt{3},-\frac{9\sqrt{3}}{2}\right)\approx(-5.2,-7.8)$

local min at $\left(3\sqrt{3},\frac{9\sqrt{3}}{2}\right)\approx(5.2,7.8)$

neither at $(0,0)$

$f''(x)=\frac{(x^2-9)^2(4x^3-54x)-(x^4-27x^2)(2)(x^2-9)(2x)}{(x^2-9)^4}$

$=\frac{(x^2-9)(4x^3-54x)-4x(x^4-27x^2)}{(x^2-9)^3}$

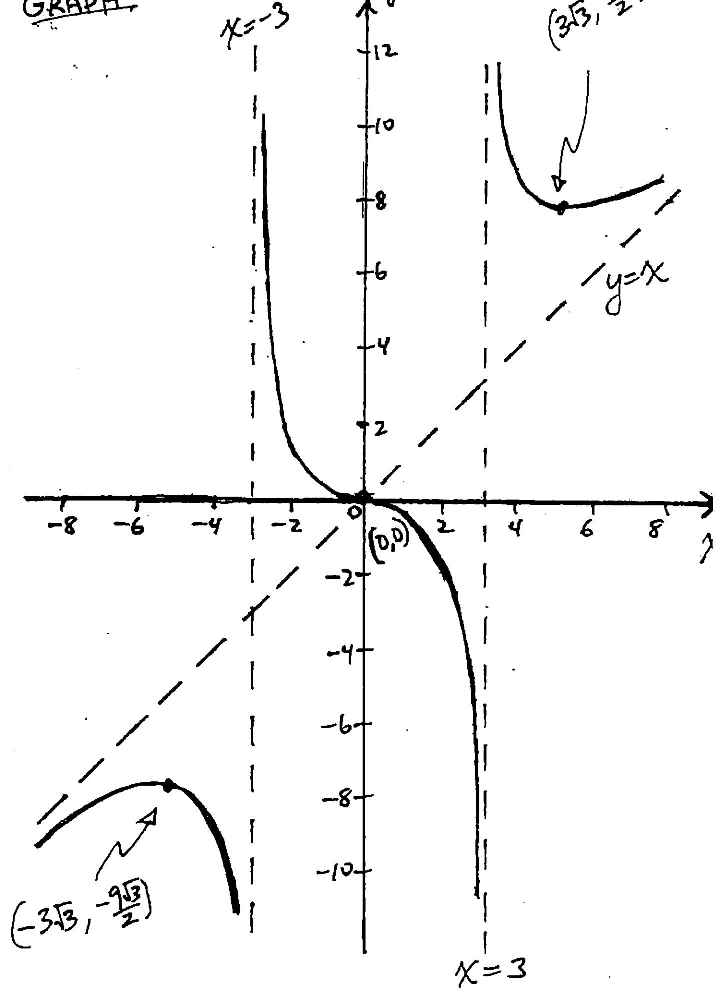
$=\frac{4x^5-90x^3+486x-4x^5+108x^3}{(x^2-9)^3}$

$=\frac{18x^3+486x}{(x^2-9)^3}=\frac{18x(x^2+27)}{(x^2-9)^3}=0$ at $x=0$

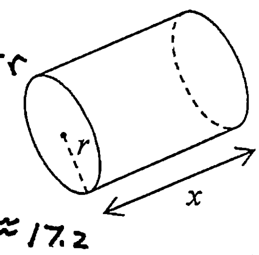
Intervals	$(-\infty,-3)$	$(-3,0)$	$(0,3)$	$(3,\infty)$
f''	−	+	−	+
f	∩	∪	∩	∪

$(0,0)$ is P.O.I.

GRAPH



Sec 3.7 #30 A cylindrical package to be sent by a postal service can have a maximum combined length and girth (circumference of a circular cross section) of 108 inches. Find the dimensions of the package of maximum volume that can be sent.



Maximize $V=\pi r^2 x$ subject to $x+2\pi r=108 \Rightarrow x=108-2\pi r$

$\therefore V=\pi r^2(108-2\pi r)=108\pi r^2-2\pi^2 r^3$

$r\geq 0$ and since $x\geq 0$ then $108-2\pi r\geq 0 \Rightarrow r\leq \frac{54}{\pi}\approx 17.2$

$\therefore$ domain is $[0,\frac{54}{\pi}]$.

$V'=216\pi r-6\pi^2 r^2=6\pi r(36-\pi r)=0$ for $r=0$ and $r=\frac{36}{\pi}$.

Any of the following verifies that V is maximized at $r=\frac{36}{\pi}$

① Check endpoints & critical #

$V(0)=0$

$V(\frac{36}{\pi})=\frac{46656}{\pi}$ ← MAX

$V(\frac{54}{\pi})=0$

② Apply FDT

	$(0,\frac{36}{\pi})$	$(\frac{36}{\pi},\frac{54}{\pi})$ or ∞
V'	+	−
V	↗	↘

$\therefore$ max at $\frac{36}{\pi}$

③ Apply SDT

$V''=216\pi-12\pi^2 r$

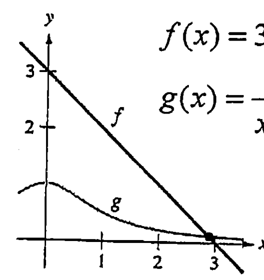
$V''(\frac{36}{\pi})=-216\pi<0$

$\therefore$ max at $\frac{36}{\pi}$

At $r=36/\pi$, $x=36$

$\therefore$ Length should be 36in and radius should be $\frac{36}{\pi}\approx 11.46$in

Sec 3.8 #18: Apply Newton's Method to approximate the x-value of the indicated point of intersection of the two graphs. Express your answer so that it is accurate to as many decimal places as your Sharp EL-531 calculator will give you. Clearly identify your starting value and list your sequence of approximations. *[Hint: Let $h(x)=f(x)-g(x)$.]*



$f(x)=3-x$

$g(x)=\frac{1}{x^2+1}$

Let $h(x)=3-x-\frac{1}{x^2+1}$. Then $h'(x)=-1+\frac{2x}{(x^2+1)^2}$

$\therefore x_{n+1}=x_n-\frac{h(x_n)}{h'(x_n)}=x_n-\left[\frac{3-x_n-\frac{1}{x_n^2+1}}{-1+\frac{2x_n}{(x_n^2+1)^2}}\right]$

for $n\geq 1$.

Let $x_1=3$. Then $x_2=2.893617021$

$x_3=2.893289200$

$x_4=2.893289196$

$x_5=2.893289196$ ← Answer.

Sec 3.9 #34: The measurements of the base and altitude of a triangle are found to be 36 and 50 centimeters, respectively. The possible error in each measurement is 0.25 centimeter. Use differentials to approximate the possible *propagated* and *percent* errors in computing the area of the triangle.

$b=36$cm, $h=50$cm, $db=\pm 0.25$cm, $dh=\pm 0.25$cm

$A=\frac{1}{2}bh=\frac{1}{2}(36)(50)=900\text{cm}^2$

$\Delta A\approx dA=\frac{1}{2}b\,dh+\frac{1}{2}h\,db=\frac{1}{2}(36)(\pm 0.25)+\frac{1}{2}(50)(\pm 0.25)$

$=\pm 10.75\text{cm}^2$ (propagated error)

$\frac{\Delta A}{A}\approx\frac{dA}{A}=\frac{10.75\text{cm}^2}{900\text{cm}^2}\approx 0.011944=1.19\%$ (percent error)

Sec 3.9 #46: Use differentials to approximate $(2.99)^3$ and compare your answer with that of your calculator.

Let $f(x)=x^3$. $f'(x)=3x^2$

$f(x+\Delta x)\approx f(x)+f'(x)dx=x^3+3x^2dx$

Let $x=3$ and $dx=\Delta x=-0.01$

Then $(2.99)^3=(3-0.01)^3\approx 3^3+3(3)^2(-0.01)=27-0.27=26.73$

From calc., $(2.99)^3=26.730899$