MATH 100 **ASSIGNMENT #3** **Name:** SOLUTIONS **Section:**___

First work through the recommended practice problems listed in the following table from the 11th edition of *Calculus of a Single Variable* by Larson and Edwards. You do not need to hand these in. Once you have completed these, then do the small sampling of questions below. Write full solutions (not just the final answer) in the space provided.

$\overline{\quad 25}$

2.5 Implicit Differentiation	5, 9, 13, 15, 17, 21, 25, 27, 29, 31, 35, 37, 39, 49, 61
2.6 Related Rates	3, 5, 7, 9, 11, 13, 15, 17, 21, 23, 25, 27, 37, 41, 43, 45
3.1 Extrema on an Interval	15, 21, 25, 27, 29, 31, 33, 37, 39, 41, 43, 57
3.2 Rolle's Theorem and the Mean Value Theorem	3, 5, 9, 11, 15, 17, 19, 23, 29, 33, 34, 37, 39, 41, 43, 45, 47, 53, 63
3.3 Increasing and Decreasing Functions and the First Derivative Test	11, 13, 17, 21, 25, 29, 31, 33, 35, 37, 41, 45, 47, 57, 59, 61, 67, 81, 95
3.4 Concavity and the Second Derivative Test	3, 4, 5, 7, 13, 15, 17, 23, 27, 33, 35, 37, 39, 41, 43, 49, 53, 55

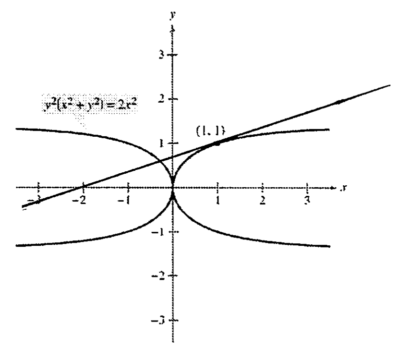
Sec 2.5 #20: Find dy/dx by implicit differentiation: $x = \sec\frac{1}{y}$

$$\frac{d}{dx}[x] = \frac{d}{dx}\left[\sec\frac{1}{y}\right]$$

$$1 = \sec\frac{1}{y}\tan\frac{1}{y}\cdot\left(-\frac{1}{y^2}\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-y^2}{\sec\frac{1}{y}\tan\frac{1}{y}} = -y^2\cos\frac{1}{y}\cot\frac{1}{y}$$

Sec 2.5 #42: **Kappa curve:** Find an equation of the tangent line to the graph of $y^2(x^2+y^2) = 2x^2$ at the point $(1, 1)$. Express your answer in the form $y = mx + b$.



$$\frac{d}{dx}\left(x^2y^2+y^4\right) = \frac{d}{dx}\left(2x^2\right)$$

$$x^2\cdot 2yy' + 2xy^2 + 4y^3y' = 4x$$

$$(2x^2y+4y^3)y' = 4x-2xy^2$$

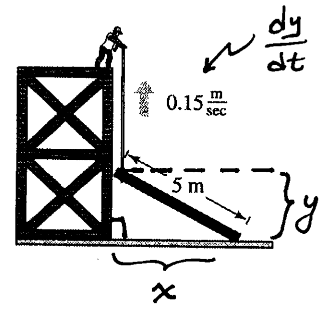
$$y' = \frac{4x-2xy^2}{2x^2y+4y^3} = \frac{2x-xy^2}{x^2y+2y^3} \text{ OR } \frac{x(2-y^2)}{y(x^2+2y^2)}$$

At $(1,1)$, $y' = \frac{1}{3}$ (slope) $\therefore$ $y-1 = \frac{1}{3}(x-1)$

$$y-1 = \frac{1}{3}x-\frac{1}{3}$$

$$y = \frac{1}{3}x+\frac{2}{3}$$

Sec 2.6 #22: **Construction:** A construction worker pulls a five-meter plank up the side of a building under construction by means of a rope tied to one end of the plank (see figure). Assume the opposite end of the plank follows a path perpendicular to the wall of the building and the worker pulls the rope at a rate of 0.15 meter per second. How fast is the end of the plank sliding along the ground when it is 2.5 meters from the wall of the building?



Given: $\frac{dy}{dt} = 0.15\ m/s$

Find: $\frac{dx}{dt}$ when $x = 2.5\ m$

Equation: $x^2+y^2 = 25 \implies y = \sqrt{25-x^2}$

$$2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{y}{x}\cdot\frac{dy}{dt} = -\frac{\sqrt{25-x^2}}{x}\cdot\frac{dy}{dt}$$

When $x = 2.5$, $\frac{dx}{dt} = -\frac{\sqrt{25-2.5^2}}{2.5}(0.15) = -\frac{\sqrt{18.75}}{2.5}(0.15) \approx -0.26\ m/s$

The end of the plank is sliding toward the wall at about $0.26\frac{m}{s}$.

Sec 3.1 #38: Find the absolute extrema of the function $g(x) = \sec x$ on the closed interval $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$.

$g'(x) = \sec x \tan x$

$g'(x)$ is defined on $[-\frac{\pi}{6}, \frac{\pi}{3}]$

$g'(x) = 0$ at $x = 0$ (critical #)

$g(-\frac{\pi}{6}) = \frac{2}{\sqrt{3}} \approx 1.15$

$g(0) = 1$ ← MIN

$g(\frac{\pi}{3}) = 2$ ← MAX

The absolute max of g is 2 (at $x = \frac{\pi}{3}$).

The absolute min of g is 1 (at $x = 0$).

Sec 3.2 #46: Determine whether the Mean Value Theorem can be applied to $f(x) = \sqrt{2-x}$ on the closed interval $[a, b] = [-7, 2]$. If the Mean Value Theorem can be applied, find all values of c in the open interval $(-7, 2)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. If the Mean Value Theorem cannot be applied, explain why not.

f is continuous and defined on $[-7, 2]$ ✓

$f'(x) = \frac{-1}{2\sqrt{2-x}}$ which exists on $(-7, 2)$ and so f is differentiable on $(-7, 2)$ ✓

∴ MVT applies: $f'(c) = \frac{f(2) - f(-7)}{2 - (-7)} = \frac{0-3}{9} = -\frac{1}{3}$

$-\frac{1}{2\sqrt{2-c}} = -\frac{1}{3} \Rightarrow 2\sqrt{2-c} = 3 \Rightarrow \sqrt{2-c} = \frac{3}{2} \Rightarrow 2 - c = \frac{9}{4}$

∴ $c = -\frac{1}{4}$

Sec 3.3 #42(a,b): Consider the function $f(x) = \sin x \cos x + 5$ on the interval $(0, 2\pi)$.

(a) Find the open interval(s) on which the function is increasing or decreasing and (b) apply the First Derivative Test to identify all relative extrema.

a) $f'(x) = \sin x(-\sin x) + \cos x(\cos x) = \cos^2 x - \sin^2 x = \cos 2x$

$f'(x) = 0$ at $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ in $(0, 2\pi)$

Interval	$(0, \frac{\pi}{4})$	$(\frac{\pi}{4}, \frac{3\pi}{4})$	$(\frac{3\pi}{4}, \frac{5\pi}{4})$	$(\frac{5\pi}{4}, \frac{7\pi}{4})$	$(\frac{7\pi}{4}, 2\pi)$
f'	+	−	+	−	+
f	↗	↘	↗	↘	↗

∴ f is increasing on $(0, \frac{\pi}{4})$, $(\frac{3\pi}{4}, \frac{5\pi}{4})$ and $(\frac{7\pi}{4}, 2\pi)$

f is decreasing on $(\frac{\pi}{4}, \frac{3\pi}{4})$ and $(\frac{5\pi}{4}, \frac{7\pi}{4})$

b) $f(\frac{\pi}{4}) = \frac{11}{2}$, $f(\frac{3\pi}{4}) = \frac{9}{2}$, $f(\frac{5\pi}{4}) = \frac{11}{2}$, $f(\frac{7\pi}{4}) = \frac{9}{2}$

By FDT, f has relative max at $(\frac{\pi}{4}, \frac{11}{2})$ and $(\frac{5\pi}{4}, \frac{11}{2})$ and relative min at $(\frac{3\pi}{4}, \frac{9}{2})$ and $(\frac{7\pi}{4}, \frac{9}{2})$.

Sec 3.4 #38: Let $f(x) = -x^4 + 2x^3 + 8x$. Find all relative extrema. Use the Second Derivative Test where applicable.

$f'(x) = -4x^3 + 6x^2 + 8 = -2(2x^3 - 3x^2 - 4)$
$= -2(x-2)(2x^2 + x + 2)$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & 0 & -4 \\ & & 4 & 2 & 4 \\ \hline & 2 & 1 & 2 & \boxed{0} \end{array}$$

$f'(x) = 0$ only at $x = 2$ (critical #)

$b^2 - 4ac = 1^2 - 4(2)(2) = -15 < 0$
can't factor (no real roots)

$f''(x) = -12x^2 + 12x$

$f''(2) = -12(2)^2 + 12(2) = -48 + 24 = -24 < 0$

$f(2) = -2^4 + 2(2)^3 + 8(2) = -16 + 16 + 16 = 16$

∴ By SDT, f has a relative max at $(2, 16)$