MATH 100 **ASSIGNMENT #2** **Name:** SOLUTIONS **Section:**____

First work through the recommended practice problems listed in the following table from the 11th edition of *Calculus of a Single Variable* by Larson and Edwards. You do not need to hand these in. Once you have completed these, then do the small sampling of questions below. Write full solutions (not just the final answer) in the space provided.

$\overline{25}$

1.5 Infinite Limits	3, 5, 7, 9, 19, 21, 25, 27, 29, 31, 33, 35, 37, 39, 41, 49, 53, 55, 61
2.1 The Derivative and the Tangent Line Problem	11, 15, 19, 21, 25, 27, 31(a), 33(a), 39, 43, 53, 69, 73, 77, 79, 85, 89
2.2 Basic Differentiation Rules and Rates of Change	7, 9, 11, 13, 15, 17, 21, 23, 25, 29, 31, 35, 39, 43, 47, 49, 57(a), 59, 79, 85, 91, 95
2.3 Product and Quotient Rules and Higher-Order Derivatives	5, 7, 9, 11, 13, 15, 19, 21, 23, 25, 27, 29, 33, 35, 39, 41, 43, 45, 47, 49, 55, 61, 65(a), 69, 73, 83, 87, 91, 97, 131
2.4 The Chain Rule	9, 11, 13, 15, 19, 25, 27, 31, 35, 37, 39, 41, 43, 45, 47, 49, 63, 69, 73(a), 77(a), 81, 85, 87, 103

Sec 1.5 #26: Find the vertical asymptotes (if any) of the graph of the function $h(x) = \dfrac{x^2-9}{x^3+3x^2-x-3}$.

$$h(x) = \frac{(x-3)(x+3)}{x^2(x+3)-(x+3)} = \frac{(x-3)(x+3)}{(x^2-1)(x+3)} = \frac{(x-3)(x+3)}{(x-1)(x+1)(x+3)} = \frac{x-3}{(x-1)(x+1)},\ x \neq -3$$

$x = 1$ and $x = -1$ are V.A. ($x = -3$ is a hole/removable discontinuity)

Sec 1.5 #46: Find the limit (if it exists). If it does not exist, then determine whether it is ∞ or $-\infty$ or neither.

$$\lim_{x\to(\pi/2)^+} \frac{-2}{\cos x} = \frac{-2}{0^-} = \infty \quad (d.n.e.)$$

Sec 2.1 #12: Find the slope of the tangent line to the graph of the function $f(x) = 5 - x^2$ at the point $(3, -4)$ by using the limit process.

$$\text{slope} = f'(3) = \lim_{\Delta x\to 0} \frac{f(3+\Delta x) - f(3)}{\Delta x} = \lim_{\Delta x\to 0} \frac{[5-(3+\Delta x)^2] - [-4]}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{5-(9+6\Delta x+(\Delta x)^2)+4}{\Delta x} = \lim_{\Delta x\to 0} \frac{-4-6\Delta x-(\Delta x)^2+4}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{\Delta x(-6-\Delta x)}{\Delta x} = \lim_{\Delta x\to 0} (-6-\Delta x) = -6$$

Sec 2.1 #28: Find the derivative by the limit process. $h(s) = -2\sqrt{s}$

$$h'(s) = \lim_{\Delta s\to 0} \frac{h(s+\Delta s) - h(s)}{\Delta s} = \lim_{\Delta s\to 0} \frac{-2\sqrt{s+\Delta s} - (-2\sqrt{s})}{\Delta s}$$

$$= \lim_{\Delta s\to 0} \frac{-2(\sqrt{s+\Delta s} - \sqrt{s})}{\Delta s} \cdot \frac{\sqrt{s+\Delta s}+\sqrt{s}}{\sqrt{s+\Delta s}+\sqrt{s}} = \lim_{\Delta s\to 0} \frac{-2(s+\Delta s - s)}{\Delta s(\sqrt{s+\Delta s}+\sqrt{s})}$$

$$= \lim_{\Delta s\to 0} \frac{-2\Delta s}{\Delta s(\sqrt{s+\Delta s}+\sqrt{s})} = \lim_{\Delta s\to 0} \frac{-2}{\sqrt{s+\Delta s}+\sqrt{s}} = \frac{-2}{\sqrt{s}+\sqrt{s}} = \frac{-2}{2\sqrt{s}} = \frac{-1}{\sqrt{s}}$$

Sec 2.1 #72: Use the alternative form of the derivative, $f'(c) = \lim\limits_{x\to c} \dfrac{f(x)-f(c)}{x-c}$, to find $f'(4)$ (if it exists) for the function $f(x) = 3/x$.

$$f'(4) = \lim_{x\to 4} \frac{f(x) - f(4)}{x-4} = \lim_{x\to 4} \frac{\frac{3}{x} - \frac{3}{4}}{x-4} = \lim_{x\to 4} \frac{3\left(\frac{4-x}{4x}\right)}{x-4}$$

$$= \lim_{x\to 4} \frac{-3(x-4)}{4x(x-4)} = \lim_{x\to 4} \frac{-3}{4x} = \frac{-3}{16}$$

Sec 2.2 #46: Find the derivative of the function $h(s) = \frac{s^5 + 2s + 6}{s^{1/3}}$. [Do not use product or quotient rules.]

$$h(s) = s^{14/3} + 2s^{2/3} + 6s^{-1/3}$$

$$h'(s) = \frac{14}{3}s^{11/3} + \frac{4}{3}s^{-1/3} - 2s^{-4/3} = \frac{2}{3}s^{-4/3}(7s^5 + 2s - 3)$$

$$= \frac{2(7s^5 + 2s - 3)}{3s^{4/3}}$$

Sec 2.2 #56(a): Find an equation of the tangent line to the graph of $y = x^3 - 3x$ at the point $(2, 2)$. Express your answer in the form $y = mx + b$.

$y' = 3x^2 - 3$ At $x = 2$, $y' = 3(2)^2 - 3 = 9$ (slope)

$$y - 2 = 9(x - 2)$$

$$y - 2 = 9x - 18$$

$$y = 9x - 16$$

Sec 2.3 #30: Find the derivative of the algebraic function $f(x) = \frac{x^2 + 5x + 6}{x^2 - 4}$.

$$f'(x) = \frac{(x^2-4)(2x+5) - (x^2+5x+6)(2x)}{(x^2-4)^2}$$

$$= \frac{2x^3 + 5x^2 - 8x - 20 - 2x^3 - 10x^2 - 12x}{(x^2-4)^2}$$

$$= \frac{-5x^2 - 20x - 20}{(x^2-4)^2} = \frac{-5(x^2+4x+4)}{(x^2-4)^2} = \frac{-5(x+2)^2}{(x+2)^2(x-2)^2} = \frac{-5}{(x-2)^2}$$

OR $f(x) = \frac{(x+2)(x+3)}{(x+2)(x-2)} = \frac{x+3}{x-2}$

$\therefore f'(x) = \frac{(x-2)(1) - (x+3)(1)}{(x-2)^2} = \frac{-5}{(x-2)^2}$

Sec 2.3 #84: The radius of a right circular cylinder is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}\sqrt{t}$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time. [Don't forget units.]

$$V = \pi r^2 h = \pi(\sqrt{t+2})^2\left(\tfrac{1}{2}\sqrt{t}\right) = \frac{\pi}{2}(t+2)t^{1/2} = \frac{\pi}{2}\left(t^{3/2} + 2t^{1/2}\right)$$

$$\frac{dV}{dt} = \frac{\pi}{2}\left(\frac{3}{2}t^{1/2} + t^{-1/2}\right) = \frac{\pi}{2}\left(\frac{3t^{1/2}}{2} + \frac{1}{t^{1/2}}\right) = \frac{\pi}{2}\left(\frac{3t+2}{2t^{1/2}}\right)$$

$$= \frac{\pi(3t+2)}{4t^{1/2}} \text{ in}^3/\text{sec}$$

Sec 2.4 #26: Find the derivative of the function $y = x^2\sqrt{16 - x^2}$.

$$y = x^2(16 - x^2)^{1/2}$$

$$y' = x^2 \cdot \tfrac{1}{2}(16-x^2)^{-1/2}(-2x) + 2x(16-x^2)^{1/2}$$

$$= \frac{-x^3}{(16-x^2)^{1/2}} + \frac{2x(16-x^2)}{(16-x^2)^{1/2}} = \frac{-x^3 + 32x - 2x^3}{(16-x^2)^{1/2}} = \frac{-3x^3 + 32x}{(16-x^2)^{1/2}}$$

OR $\frac{-x(3x^2 - 32)}{(16-x^2)^{1/2}}$

Sec 2.4 #48: Find the derivative of the trigonometric function $h(t) = 2\cot^2(\pi t + 2)$.

$$h'(t) = 4\cot(\pi t + 2) \cdot \left(-\csc^2(\pi t + 2)\right) \cdot \pi$$

$$= -4\pi \cot(\pi t + 2)\csc^2(\pi t + 2)$$