MATH 100 **ASSIGNMENT #1** **Name:** SOLUTIONS **Section:**

25

First work through the recommended practice problems listed in the following table from the 11th edition of *Calculus of a Single Variable* by Larson and Edwards. You do not need to hand these in. Once you have completed these, then do the small sampling of questions below. Write full solutions (not just the final answer) in the space provided.

P.1 Graphs and Models	3, 4, 5, 6, 23, 25, 31, 37, 43, 47, 57, 61
P.2 Linear Models and Rates of Change	11, 17, 23, 31, 45, 47, 57, 59, 65
P.3 Functions and Their Graphs	11, 19, 23, 29, 35, 41, 43, 49, 53, 55, 63, 75
P.4 Review of Trigonometric Functions	9, 11, 19, 23, 31, 35, 37, 41, 45, 57, 65
1.1 A Preview of Calculus	7, 9
1.2 Finding Limits Graphically and Numerically	7, 17, 21, 23, 25, 27, 29, 31, 33, 45, 47, 73, 75
1.3 Evaluating Limits Analytically	7, 11, 15, 21, 29, 35, 45, 47, 51, 53, 57, 61, 63, 65, 73, 87, 89, 105, 115, 119, 121
1.4 Continuity and One-Sided Limits	5, 7, 9, 13, 15, 17, 23, 31, 35, 37, 41, 43, 47, 49, 51, 61, 67, 77, 81, 83, 85, 95, 105

Sec P.1 #48: Find any intercepts, test for symmetry about each axis and the origin, and then sketch the graph of the equation $y=\sqrt{25-x^2}$. As always, number and label your axes.

Semicircle $x^2+y^2=25,\ y\geq 0$ with center $(0,0)$ and radius 5

y-int: $y=\sqrt{25-0^2}=5$, $(0,5)$

x-int: $\sqrt{25-x^2}=0$

$25-x^2=0$

$x^2=25$

$x=\pm 5$

$\therefore (5,0), (-5,0)$

Symmetry Tests

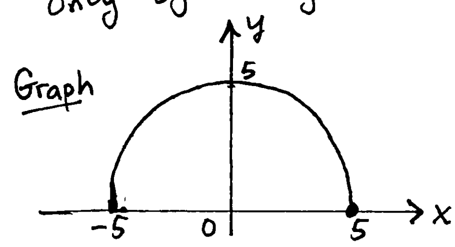
(i) y-axis: $y=\sqrt{25-(-x)^2}$, $y=\sqrt{25-x^2}$ ✓

(ii) x-axis: $-y=\sqrt{25-x^2}$, $y=-\sqrt{25-x^2}$ ✗

(iii) origin: $-y=\sqrt{25-(-x)^2}$, $y=-\sqrt{25-x^2}$ ✗

Only symmetry about y-axis

Graph



Sec P.2 #62: Write the general form $Ax+By=C$ (where A, B and C are integers and $A>0$) of the equation of the line through the point $(5/6, -1/2)$ (a) parallel and (b) perpendicular to the line $7x+4y=8$.

$\to y=-\frac{7}{4}x+2$, slope $-\frac{7}{4}$

a) $m=-\frac{7}{4}$

$y+\frac{1}{2}=-\frac{7}{4}\left(x-\frac{5}{6}\right)$

$y+\frac{1}{2}=-\frac{7}{4}x+\frac{35}{24}$

$y=-\frac{7}{4}x+\frac{23}{24}$

$\to 24y=-42x+23$

$42x+24y=23$

b) $m=\frac{4}{7}$

$y+\frac{1}{2}=\frac{4}{7}\left(x-\frac{5}{6}\right)$

$y+\frac{1}{2}=\frac{4}{7}x-\frac{10}{21}$

$y=\frac{4}{7}x-\frac{41}{42}$

$\to 42y=24x-41$

$24x-42y=41$

Sec P.3 #46: Determine whether y is a function of x: $x^2y-x^2+4y=0$

$y(x^2+4)=x^2$

$y=\frac{x^2}{x^2+4}$

y is a function of x since there is only one y-value associated with each x-value.

Sec P.4 #38: Solve the equation for θ, where $0\leq\theta\leq 2\pi$: $2\cos^2\theta-\cos\theta=1$

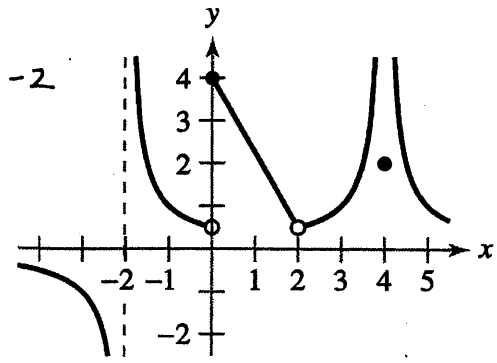
$2\cos^2\theta-\cos\theta-1=0$

$(2\cos\theta+1)(\cos\theta-1)=0$

$\cos\theta=-\frac{1}{2}$ or $\cos\theta=1$

$\theta=\frac{2\pi}{3}, \frac{4\pi}{3}, 0, 2\pi$

Sec 1.2 #30: Use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not, then write "dne" (for "does not exist") and briefly explain why it doesn't exist.



(a) $f(-2) =$ dne -2 is not in domain of f (i.e. f is not defined at -2)

(b) $\lim_{x\to -2} f(x) =$ dne f is unbounded as x approaches -2

(c) $f(0) = 4$

(d) $\lim_{x\to 0} f(x) =$ dne f approaches different values on either side of 0

(e) $f(2) =$ dne 2 is not in domain of f

(f) $\lim_{x\to 2} f(x) = \frac{1}{2}$

(g) $f(4) = 2$

(h) $\lim_{x\to 4} f(x) =$ dne f is unbounded as x approaches 4

Sec 1.2 #48: Find the limit L. Then use the ε - δ definition to prove that the limit is L. $\lim_{x\to 3}(\frac{3}{4}x+1)$

$= \frac{3}{4}(3)+1 = \frac{9}{4}+1 = \frac{13}{4}$

$\therefore L = \frac{13}{4}$

Proof Let $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$

$|(\frac{3}{4}x+1) - \frac{13}{4}| < \varepsilon$

$|\frac{3}{4}x - \frac{9}{4}| < \varepsilon$

$\frac{3}{4}|x-3| < \varepsilon$

$|x-3| < \frac{4}{3}\varepsilon$

$\therefore$ Let $\delta = \frac{4}{3}\varepsilon$

Suppose $0 < |x-3| < \delta$ where $\delta = \frac{4}{3}\varepsilon$. Then

$$|f(x)-L| = |(\tfrac{3}{4}x+1) - \tfrac{13}{4}| = |\tfrac{3}{4}x - \tfrac{9}{4}| = \tfrac{3}{4}|x-3| < \tfrac{3}{4}\cdot\delta = \tfrac{3}{4}\cdot\tfrac{4}{3}\varepsilon = \varepsilon,$$

which proves $\lim_{x\to 3}(\frac{3}{4}x+1) = \frac{13}{4}$.

Sec 1.3 #54: Find the limit (if it exists).

$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x\to 3}\frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3}\frac{x-3}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{3+1}+2} = \frac{1}{4}$$

Sec 1.4 #24: Find the one-sided limit $\lim_{x\to 1^+} f(x)$ (if it exists), where $f(x) = \begin{cases} x, & x \le 1 \\ 1-x, & x > 1 \end{cases}$.

If it does not exist, explain why.

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(1-x) = 1-1 = 0$$

Sec 1.4 #100: Let $f(x) = \dfrac{x^2+x}{x-1}$ on the interval $[5/2, 4]$ and let $k = 6$. Verify that the conditions of the Intermediate Value Theorem are satisfied and then find the value(s) of c in the given interval for which $f(c) = 6$ as guaranteed to exist by the theorem.

f is continuous on $[5/2, 4]$

f has a discontinuity at $x = 1$ but $1 \notin [5/2, 4]$.

$f(5/2) = \frac{35}{6} \approx 5.83 < 6$

$f(4) = \frac{20}{3} \approx 6.67 > 6$

$\therefore$ Conditions of IVT are satisfied

$f(x) = 6$

$\frac{x^2+x}{x-1} = 6$

$x^2+x = 6x-6$

$x^2-5x+6 = 0$

$(x-3)(x-2) = 0$

$x = 3$ or $\underbrace{x=2}_{\text{not in interval}}$

$\therefore c = 3$